Elementary-Level Mathematics

Use of different meanings of addition, subtraction, multiplication and division

Québec

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Table of Contents

Introduction

In order to develop a good understanding of operations and their different meanings, students will be encouraged to mathematize a variety of situations.

In these situations, students will learn to break problems down into simpler ones and identify the relationships among data that will help them to arrive at a solution. Since operation sense is developed at the same time as number sense, the two should be taught concurrently[1].

In addition and subtraction, quantities are added, taken away, united or compared. It is important for students to work on all the types of problems to make use of the different meanings of addition and subtraction.

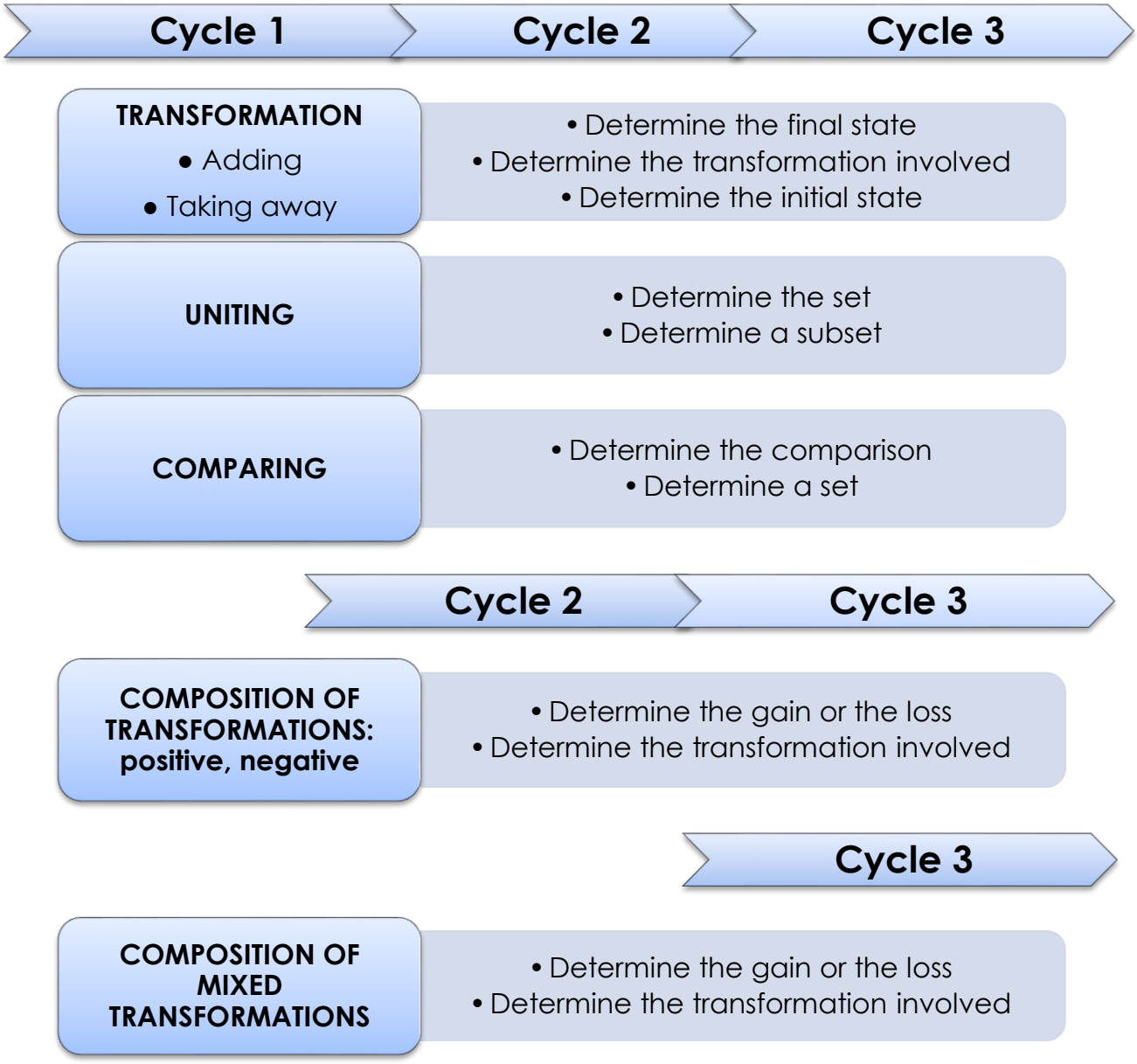
To develop an understanding of multiplication and division, students must identify the three underlying quantities: the total quantity, the number of equal groups and the size of each group. In this case as well, teachers must assign students all the types of problems involving the different meanings of multiplication and division.

[1] Ministère de l'Éducation et de l'Enseignement supérieur, *Progression of Learning in Elementary School – Mathematics*, Québec, 2009, 9.

Situations involving additive structures

Techniques for performing operations, relationships between operations and the properties of operations only have real meaning when they are used to mathematize situations in order to solve problems. Additive structures deal with addition and subtraction. To enable students to explore different structures, it is essential that a variety of situations be presented: transformation (adding or taking away), uniting, comparing (more or fewer than) and composition of transformations (positive, negative or mixed).

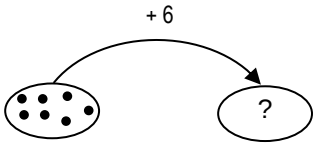
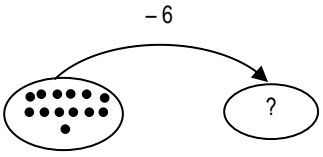
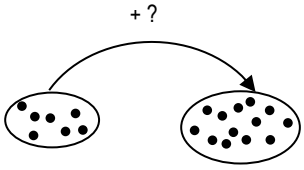
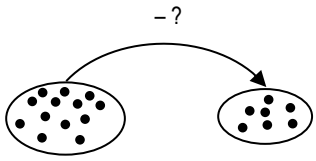
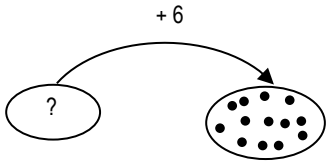
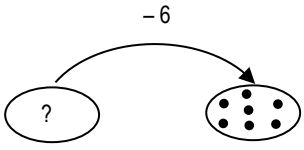
Students do not have to know or memorize the different names for these structures. They must instead develop their own ways of representing these structures to give them meaning.



The situations presented on the following pages illustrate the various meanings of addition and subtraction.

Transformation

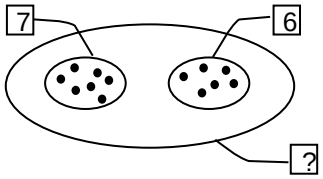
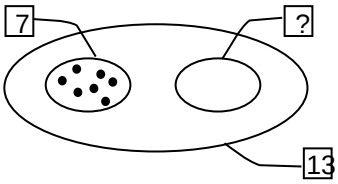
Cycle 1 → **Cycle 2** → **Cycle 3**

Structure		Situation[2]	Model (Students create their own representations according to the situation)	Equation
Determine the final state	**ADDING**	Gus had 7 objects. Melanie gave him 6 more. How many objects does Gus now have?		7 + 6 = ❒
	TAKING AWAY	Gus had 13 objects. He gave 6 of them to Melanie. How many objects does Gus now have?		13 – 6 = ❒
Determine the transformation involved	**ADDING**	Gus had 7 objects. Melanie gave him some objects. Gus now has 13 objects. How many objects did Melanie give Gus?		7 + ❒ = 13
	TAKING AWAY	Gus had 13 objects. He gave some to Melanie. Gus now has 7 objects. How many objects did Gus give Melanie?		13 – ❒ = 7
Determine the initial state	**ADDING**	Gus had some objects. Melanie gave him 6 more. Gus now has 13 objects. How many objects did Gus start with?		❒ + 6 = 13
	TAKING AWAY	Gus had a certain number of objects. He gave 6 to Melanie. He now has 7 objects. How many objects did Gus start with?		❒ – 6 = 7

[2] The following examples consist of only two values each. Teachers should make sure to present situations consisting of several values, as well as superfluous or missing data.

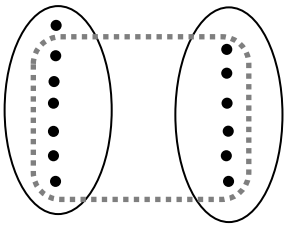
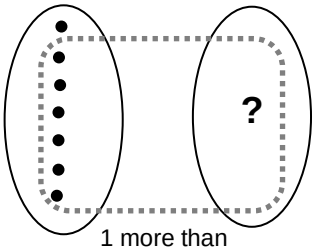
Uniting

Cycle 1 → Cycle 2 → Cycle 3

Structure	Situation[3]	MODEL (Students create their own representations according to the situation)	Equation
Determine the set	Gus has 7 objects. Melanie has 6. How many objects do they have in all?		7 + 6 = ❒
Determine a subset (complement)	Melanie and Gus have 13 objects all together. Gus has 7. How many objects does Melanie have?		7 + ❒ = 13 13 – 7 = ❒

Comparing

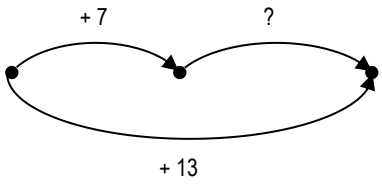
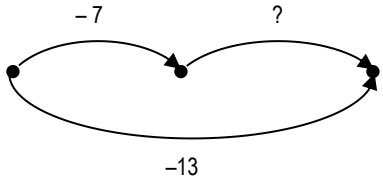
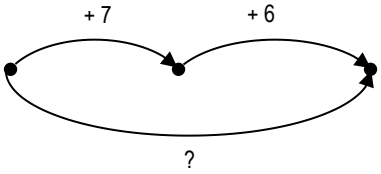
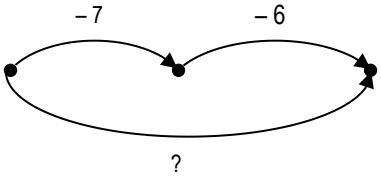
Cycle 1 → Cycle 2 → Cycle 3

Structure		Situation[3]	MODEL (Students create their own representations according to the situation)	Equation
Determine the comparison	**MORE THAN**	Gus has 7 objects. Melanie has 6. How many more objects than Melanie does Gus have?	**?** more than **?** less than 	7 = 6 + ❒ 7 – ❒ = 6
	FEWER THAN	Gus has 7 objects. Melanie has 6. How many fewer objects than Gus does Melanie have?		7 = 6 + ❒ 7 – ❒ = 6
Determine a set	**MORE THAN**	Gus has 7 objects. He has 1 more object than Melanie. How many objects does Melanie have?	 1 more than 1 less than	7 – 1 = ❒ 7 = ❒ + 1
	FEWER THAN	Gus has 7 objects. Melanie has 1 fewer object than Gus. How many objects does Melanie have?		7 – 1 = ❒ 7 = ❒ + 1

[3] The following examples consist of only two values each. Teachers should make sure to present situations consisting of several values, as well as superfluous or missing data.

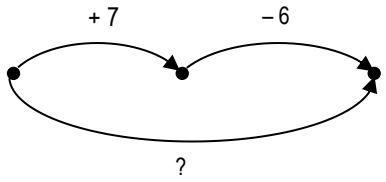
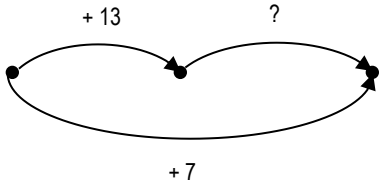
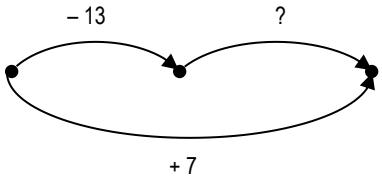
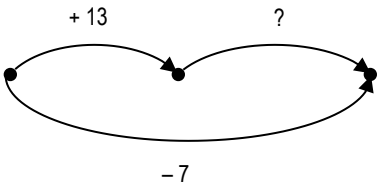
Composition of transformations: positive, negative

Cycle 2 | Cycle 3

Structure		Situation	MODEL (Students create their own representations according to the situation)	Equation
Determine the transformation involved	**POSITIVE**	Yesterday, Gus received 7 objects. Today, he has received more but we do not know how many. Given that he has received 13 objects in the past 2 days, how many more or fewer objects does he have today?		7 + ❒ = 13
	NEGATIVE	Yesterday Gus gave away 7 objects. Today, he has given away some more, but we do not know how many. If in the past 2 days he has given away 13 objects, how many objects has he given away today?		7 + ❒ = 13
Determine the gain or the loss	**POSITIVE**	Yesterday, Gus received 7 objects. Today, he has received 6 more. How many objects has he received over the 2 days?		7 + 6 = ❒
	NEGATIVE	Yesterday, Gus gave away 7 objects. Today, he has given away 6. How many objects has he given away in the past 2 days?		7 + 6 = ❒

Composition of mixed transformations[4]

Cycle 3

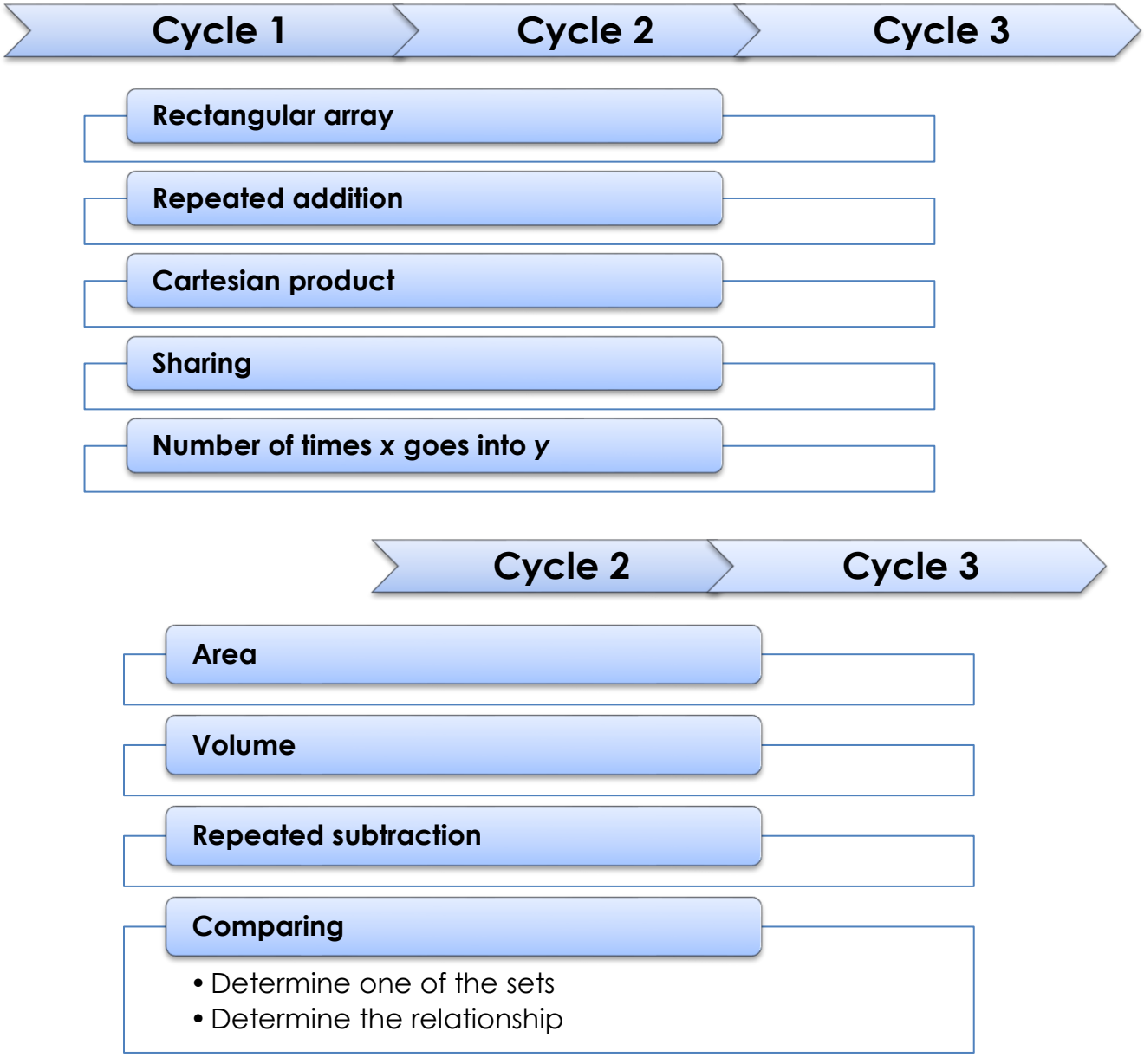
Structure	Situation	MODEL (Students create their own representations according to the situation)	Equation
Determine the gain or the loss	Gus played a card game. He won 7 points in the first hand. He lost 6 points in the second hand. In all, how many points did he win or lose during these two hands?		7 – 6 = ❐
Determine the transformation involved	Gus played a card game. He won 13 points in the first hand. We do not know how he did in the second hand, but we know that he won a total of 7 points after two hands. How many points did he win or lose in the second hand?		13 – ❐ = 7
	Gus played a card game. He lost 13 points in the first hand. We do not know how he did in the second hand, but we know that he won a total of 7 points after two hands. How many points did he win or lose in the second hand?		See footnote
	Gus played a card game. He won 13 points in the first hand. We do not know how he did in the second hand, but we know that he lost a total of 7 points after two hands. How many points did he win or lose in the second hand?		

[4] Problems involving a composition of mixed transformations require the use of integers. These problems are solved using a diagram or a number line.

Situations involving multiplicative structures

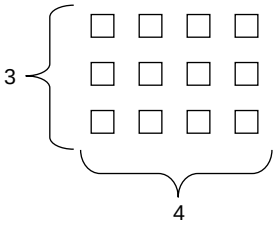
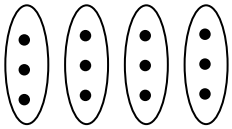
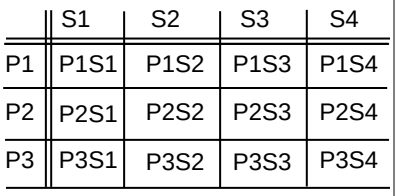
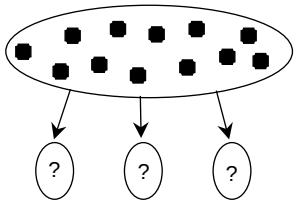
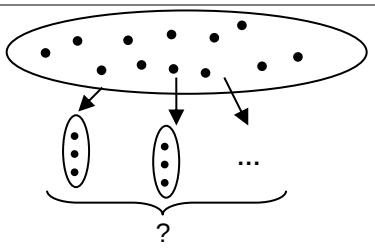
Techniques for performing operations, relationships between operations and the properties of operations only have real meaning when they are used to mathematize situations in order to solve problems. Multiplicative structures deal with multiplication and division. To enable students to explore different structures, it is essential that a variety of situations be presented: repeated addition, combination or Cartesian product, rectangular array, area and volume, comparison (times as many), repeated subtraction, sharing, number of times *x* goes into *y*, comparison (times fewer than).

Students do not have to know or memorize the different names for these structures. They must instead develop their own ways of representing these structures to give them meaning.

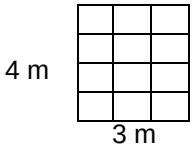
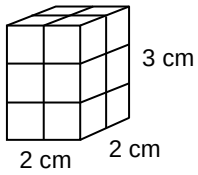
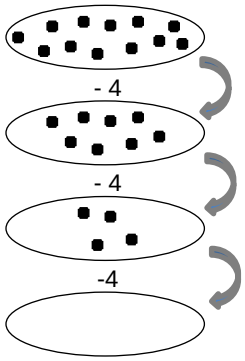
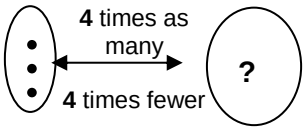
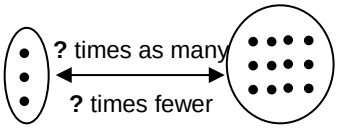


The situations presented on the following pages illustrate the various meanings of multiplication and division.

Cycle 1 → **Cycle 2** → **Cycle 3**

Structure	Situation	MODEL (Students create their own representations according to the situation)	Equation
Rectangular array	In the classroom, there are 3 rows containing 4 desks each. How many desks are in the classroom?		$3 \times 4 = \square$ or $4 \times 3 = \square$
Repeated addition	Gus receives 3 objects per day. How many objects will he receive in 4 days?		$3 + 3 + 3 + 3 = \square$ $4 \times 3 = \square$ or $3 \times 4 = \square$
Cartesian product	Gus has 4 shirts and 3 pairs of pants. How many combinations of pants and shirts can he wear?		$4 \times 3 = \square$ or $3 \times 4 = \square$
Sharing	There are 12 objects in a bag. They are distributed equally among 3 friends. How many objects does each friend get?		$12 \div 3 = \square$
Number of times *x* goes into *y*	12 objects must be placed in bags. Each bag can hold 3 objects. How many bags are needed?		$12 \div 3 = \square$

Cycle 2 → Cycle 3

Structure		Situation	MODEL (Students create their own representations according to the situation)	Equation
Area		A flower bed is 3 m wide and 4 m long. What is the area of this flower bed?		$4 \times 3 = \square$ or $3 \times 4 = \square$
Volume		A box in the shape of a rectangular prism is 2 cm wide, 2 cm deep and 3 cm high. What is the volume of the box?		$2 \times 2 \times 3 = \square$ $2 \times 3 \times 2 = \square$ or …
Repeated subtraction		Gus has 12 marbles. He would like to give 4 to each of his friends. How many friends will receive 4 marbles?		12 - 4 8 8 - 4 4 4 - 4 0
Comparing: **Determine one of the sets**	**TIMES AS MANY**	Gus has 3 objects. Melanie has 4 times as many objects. How many objects does Melanie have?		$3 \times 4 = \square$
	TIMES FEWER THAN	Melanie has 12 objects. Gus has 4 times fewer objects than Melanie. How many objects does Gus have?		$12 \div 4 = \square$
Comparing: **Determine the relationship**	**TIMES AS MANY**	Gus has 3 objects and Melanie has 12 objects. Melanie has how many times more objects than Gus?		$3 \times \square = 12$ or $12 \div 3 = \square$
	TIMES FEWER THAN	Gus has 3 objects and Melanie has 12 objects. Gus has how many times fewer objects than Melanie?		$12 \div \square = 3$ or $12 \div 3 = \square$

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