Mathematical Reasoning

Focus of Competency 2

Direction de la formation générale des jeunes
Secteur de l’éducation préscolaire et de l’enseignement primaire et secondaire
Ministère de l’Éducation et de l’Enseignement supérieur

ENSEMBLE on fait avancer le Québec

Québec

Focus of the Competency

Using mathematical reasoning involves **making conjectures and criticizing, justifying or refuting a proposition by applying an organized body of mathematical knowledge**.

Québec Education Program (QEP), Secondary Cycle One, p. 200.

By the end of Secondary Cycle One,
students should be able to . . .

Statements considered to be true even though they have not been proven

- Define a situation and propose **conjectures.**
- Apply concepts and processes appropriate to the situation.

Reject the statement

- Try different approaches in order to determine whether they should confirm or **refute** their conjectures. They validate them either by basing each step of their solution on concepts, processes, rules or statements that they express in an organized manner, or by supplying counterexamples.

QEP, Cycle One, p. 203.

By the end of Secondary Cycle two, students in all three options should be able to . . .

- make conjectures, apply appropriate concepts and processes and confirm or **refute** their conjectures by using various types of reasoning

Reject the statement

- validate conjectures by basing each step in their **proof** on concepts, processes, rules or postulates, which they express in an organized manner

Set of justifications based on observations, definitions and theorems

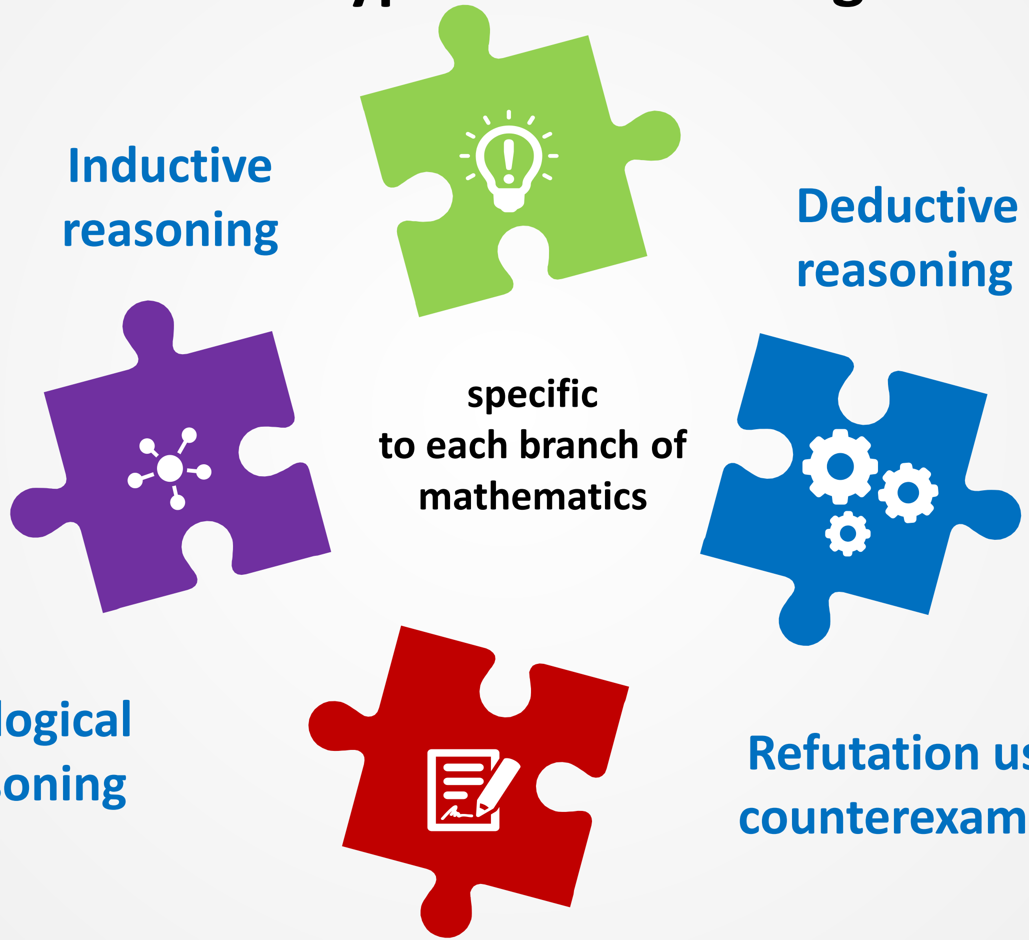
QEP, Secondary Cycle Two, Mathematics, p. 30.

Main Types of Reasoning

The **types of reasoning specific to each branch of mathematics** are arithmetic, proportional, algebraic, geometric, probabilistic and statistical reasoning.

Inductive reasoning

Deductive reasoning



Analogical reasoning

Refutation using counterexamples

Inductive Reasoning

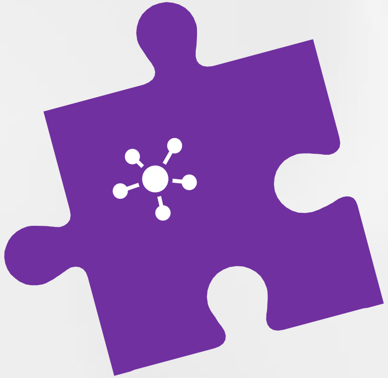
Inductive reasoning involves generalizing on the basis of individual cases.

QEP, Cycle Two, p. 26.

Analogical reasoning

Analogical reasoning involves making comparisons based on similarities in order to draw conclusions [or make conjectures].

QEP, Cycle Two, p. 26.



Deductive Reasoning

Deductive reasoning involves a [logical] series of propositions that lead to conclusions based on principles that are considered to be true.

QEP, Cycle Two, p. 26.

Refutation Using Counterexamples

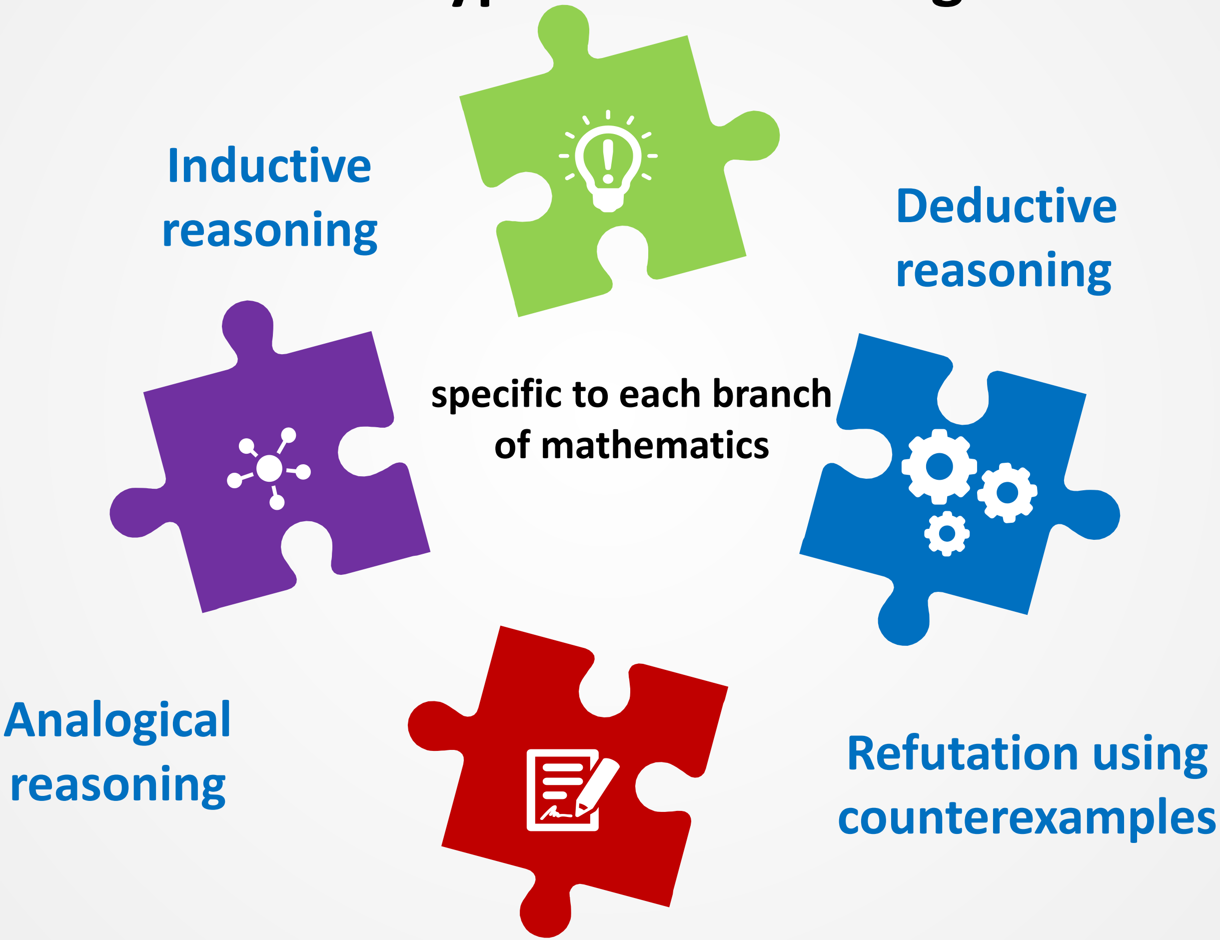
Refutation using counterexamples involves disproving a conjecture without stating what is true.

QEP, Cycle Two, p. 26.

- Only one counterexample is required to show that a conjecture is false.
- One cannot conclude that a mathematical statement is true simply because several examples show it to be true.

QEP, Cycle One, p. 201.

Main Types of Reasoning



Strategies for Exercising the Competencies

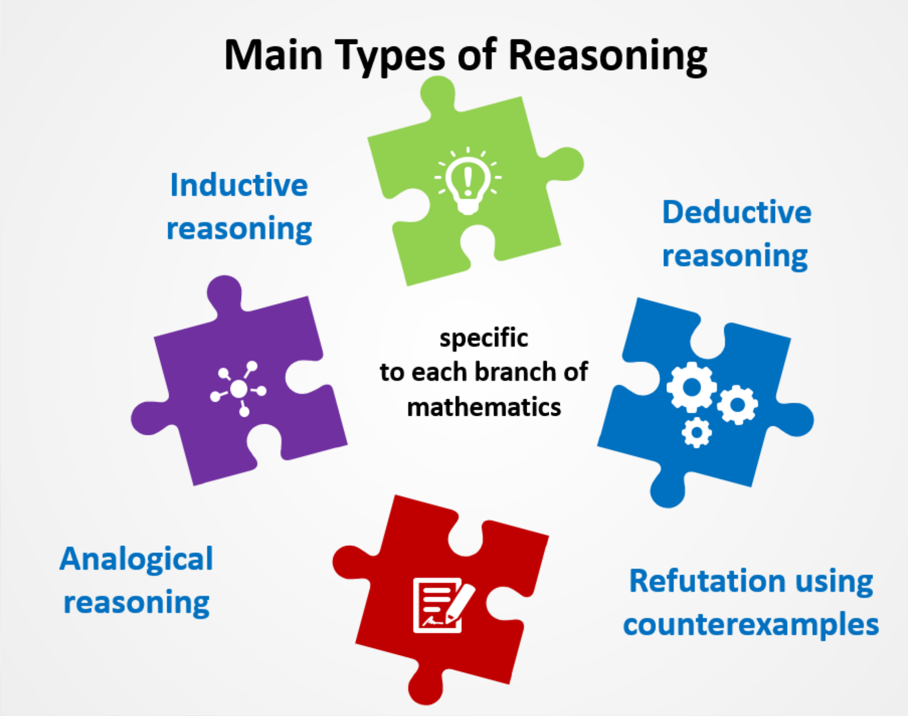
- Representing the situation mentally or in writing
- Giving examples
- Finding patterns
- Anticipating and interpreting results in light of the context
- Referring to a similar problem that has already been solved
- Deriving new data from known data

QEP, Cycle One, p. 220.
QEP, Cycle Two, p. 111-112.

Strategies for Exercising the Competencies

- Comparing and questioning one's procedures and results with those of the teacher or one's peers
- Understanding definitions, properties and theorems in order to use them in other contexts
- Analyzing examples; making comparisons; identifying differences and similarities
- Assessing the relevance of qualitative or quantitative data
- Etc.

QEP, Cycle One, p. 220.
QEP, Cycle Two, p. 111-112.



Reasoning and Learning

Distinction

Calculate the area of a circle whose radius is:

a) 3 cm
b) 6 cm

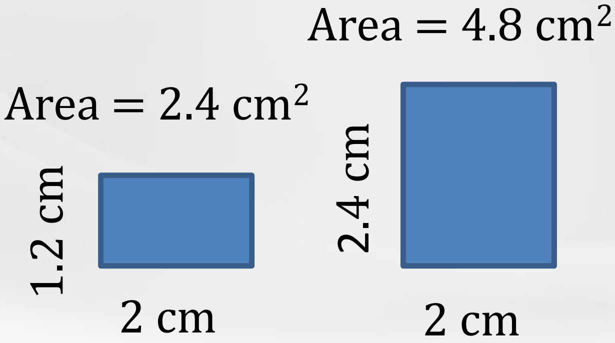
Exercise involving applications

What happens to the area of a circle if its radius is doubled?

Reasoning task

Example of preparatory questions

What happens to the area of a rectangle if its height is doubled?



Are the students' results consistent with the proposed relationship?

Do all these examples provide sufficient justification for stating that if the height doubles, the area also doubles?

Example of a justification to accompany the conjecture

The area of a rectangle can be determined by multiplying the measure of its base by its height ($A = b \times h$).

If the initial height is doubled, the area of the new rectangle is determined by multiplying the measure of its base, which is still the same, by the new height, which is the initial height multiplied by 2.

This is equivalent to multiplying the area of the initial rectangle by 2, which is why the area will be twice as large.

What happens to the area of a circle if its radius is doubled?

Various formulations

What happens to the area of a circle if its radius is doubled?

Confirm or refute the following statement: When the radius of a circle is doubled, the area of the circle also doubles.

EXAMPLES OF SITUATIONS

Examples: Cycle One

- Describe what happens to the perimeter of a rectangle when its dimensions are doubled, tripled or quadrupled.
- How is it possible to obtain a unit fraction by subtracting one unit fraction from another unit fraction?
- In the Cartesian plane, what is the geometric relationship between the points whose x-coordinate and y-coordinate add up to 5?

Examples: Cycle One (*cont.*)

- **Show** that the sum of the measures of the exterior angles of a triangle is 360º.

Prove by using rigorous reasoning based on properties, definitions and justifications

- Is the following statement true or false?
  In a statistical distribution, when the value of each data item is doubled, the mean also doubles.

Examples: Cycle One (*cont.*)

Use a proof to verify that the statement is true

- **Confirm** or **refute** the following statement: When two opposite numbers are added to a statistical distribution, the mean does not change.

Find a counterexample

- A hamburger consists of bread, tomatoes, lettuce and meat. If the price of each of these ingredients increases by 5%, by what percentage will the total price of the hamburger increase?

Examples: Cycle One (*cont.*)

- Choose two integers greater than zero. Then, determine their greatest common divisor (GCD) and their least common multiple (LCM). What can you say about the product of the GCD and the LCM of these two numbers?

- How many solutions does the equation $x^2 = 36$ have?

Examples: Secondary III

- ➔ Is the following statement true or false? More teams of 3 people than teams of 9 people can be formed from a group of 12 people.

- ➔ Show that the expressions $(x+y)^2$ and $\frac{4x^3+8x^2+4xy^2}{4x}$ are equivalent if $x \neq 0$.

Examples: Secondary III (*cont.*)

- Confirm or refute the following statement: When the water in a cylindrical container is emptied at a constant rate, the relationship between the height of the water in the cylinder and the remaining volume of water corresponds to a first-degree function.

- In a right triangle, an altitude is drawn from the vertex of the right angle. What is the relationship between the lengths of the legs, the length of the hypotenuse and the length of the altitude drawn? Explain.

Example: Secondary III (*cont.*)

- ➔ Why does the direction of the inequality symbol ($<$, $>$, $\leq$ and $\geq$) change when the terms of an inequality are multiplied or divided by a negative number?

Examples: Secondary IV

- ➔ What conjecture can you make concerning the sine of two supplementary angles?

- ➔ Is the following statement true or false? All inverses of functions are functions.

  If the statement is true, provide a proof. If it is false, provide a counterexample.

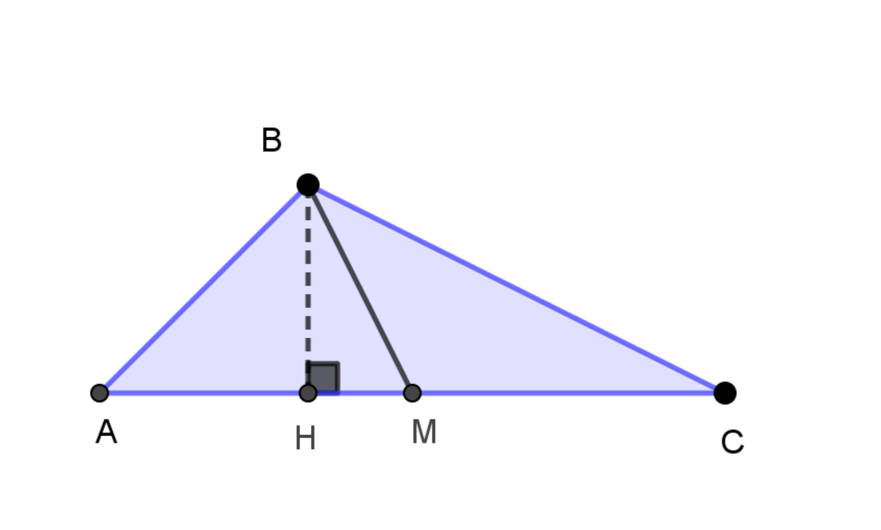
Examples: Secondary IV (*cont.*)

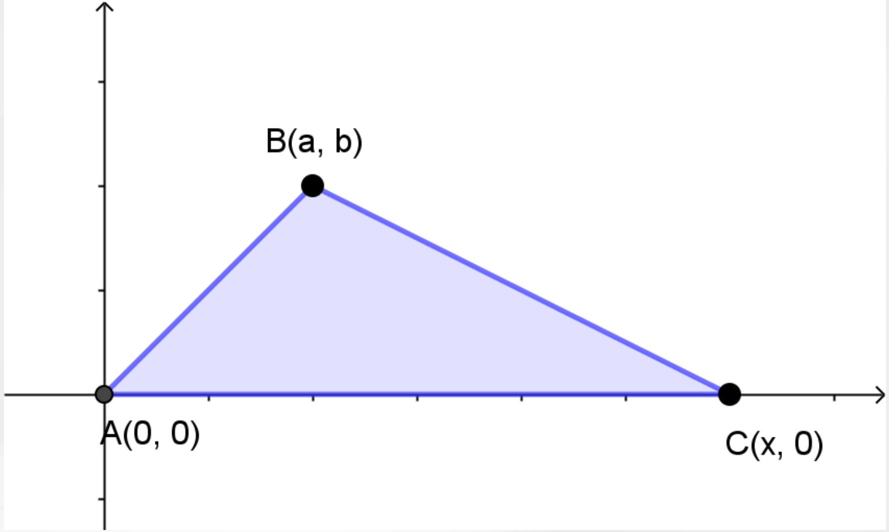
➔ Confirm or refute the following statement: Two statistical distributions with the same mean deviation have the same mean.

➔ Prove the following statements:
- In a circle, two congruent central angles have congruent chords.
- Any straight line that intersects two sides of a triangle and is parallel to the third side divides the two sides into segments of proportional lengths.

Example: Secondary IV (*cont.*)

➔ Show that the median of a triangle divides the triangle into two triangles with the same area.



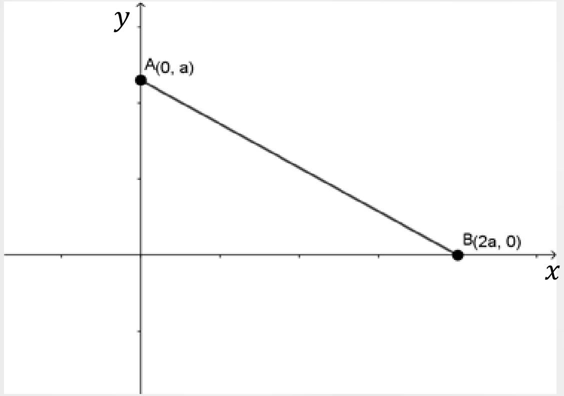


Examples: Secondary IV (*cont.*)

➔ In the Cartesian plane below, a series of line segments is drawn parallel to $\overline{AB}$. The endpoints of these segments are located on each of the two axes.

Then, the coordinates of the midpoint of each of these segments is determined.

What is the relationship between the geometric locus of the midpoints and the segments? Justify your answer.



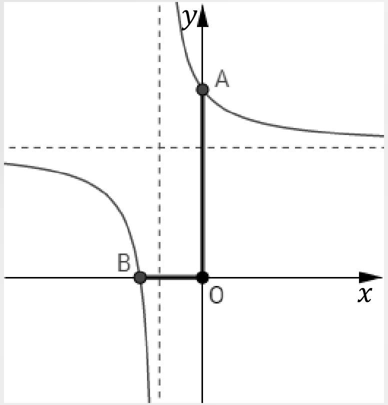
Examples: Secondary V

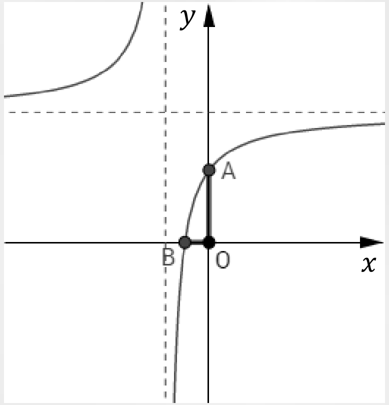
- Show that a triangle inscribed in a circle is a right triangle if one of its sides passes through the centre of the circle.

- Prove that the opposite angles of a quadrilateral inscribed in a circle are supplementary.

Examples: Secondary V (*cont.*)

➔ These are two possible graphical representations of a rational function:





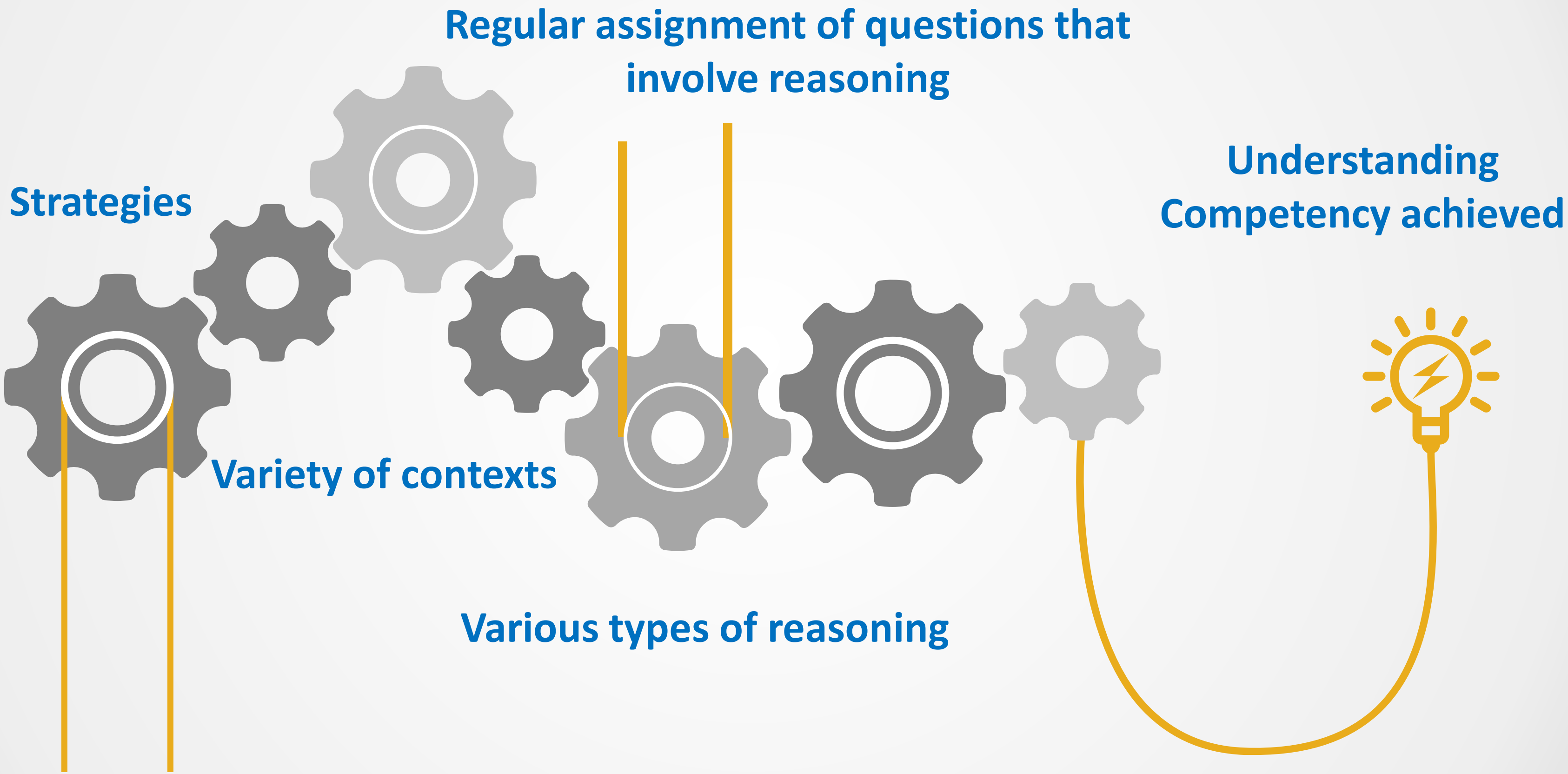
Formulate a conjecture

Make a conjecture about the relationship between the ratio $\frac{m\overline{AO}}{m\overline{BO}}$ and parameters **h** and **k** of the rule of the function.

Examples: Secondary V (*cont.*)

- ➔ What relationship can you establish between the area of the triangle formed by the graph of an absolute value function and the *x*-axis and parameters **a** and **k** of the rule of the function? Justify your answer.

Conclusion



Regular assignment of questions that involve reasoning
Strategies
Understanding Competency achieved
Variety of contexts
Various types of reasoning