Probability : Instructional Suggestions for Elementary School Teachers

Excerpt from page 21 of the Progression of Learning in Elementary School: Mathematics, Québec Education Program :

When attempting to determine the probability of an event, students in elementary school spontaneously rely on intuitive, yet often arbitrary, reasoning. Their predictions may be based on emotions, which may cause them to wish for a predicted outcome or to refute actual results. The classroom activities suggested should help foster probabilistic reasoning. This implies taking into account the uncertainty of outcomes, which may represent a challenge of sorts, since students will tend to determine outcomes by looking for patterns or expecting outcomes to balance out. (For example, if the pointer on a two-coloured spinner (red and yellow) stops on yellow three times, students will expect it to stop on red when it's their turn.)

In elementary school, students observe and conduct experiments involving chance. They use qualitative reasoning to practise predicting outcomes by becoming familiar with concepts of certainty, possibility and impossibility. They also practise comparing experiments to determine events that are more likely, just as likely and less likely to occur. They list the outcomes of a random experiment using tables or tree diagrams and use quantitative reasoning to compare the actual frequency of outcomes with known theoretical probabilities.

Introduce probability

We are surrounded by phenomena that cannot be predicted with certainty and our students surely hear expressions related to probability without necessarily noticing and/or understanding what is being addressed. Weather forecasts and games of chance are two areas where probability is used to a great extent. It would also be useful to talk about applications of probability in everyday life. This way, students will become aware of how this branch of mathematics helps them to have a better understanding of the world around them and to participate in it as active and informed citizens. It would also be interesting to validate what they have learned outside school to see what impressions, good and bad, could influence their learning.[1]

In all cycles, in order to help students acquire an understanding of chance, it is recommended that teachers use situations that help students to realize that results vary, to identify certain trends and analyze them in order to find explanations. This is a good way to encourage the development of their probabilistic thinking. Furthermore, using a topic that is meaningful to them and with which they can experiment, cannot but help improve their conceptions of chance.[2] For example, to help

[1] Laurie H. Rubel, "Students' Probabilistic Thinking Revealed: The Case of Coin Tosses." *Thinking and Reasoning with Data and Chance*, *68th Yearbook*. (Reston: The National Council of Teachers of Mathematics, 2006), 49.

[2] Rubel, "Students' Probabilistic Thinking Revealed," 51.

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a student understand that the probability of turning up one head and one tail when tossing two coins is not 1 out of 3, you toss two dice. You then ask the student if it is more likely that the sum of the numbers on the dice will be 11 or will be 12. The student should be able to answer that 11 (5 and 6 or 6 and 5) is more likely than 12 (6 and 6). Taking the coins again, the student can realize that there is a greater likelihood of obtaining one head and one tail than of obtaining two heads or two tails and, therefore, that the probability cannot be 1 out of 3. Tables or diagrams can make it easier to represent and explain these phenomena, by enabling the student to get a better grasp of what they have in common.

If you get small groups to experiment with activities related to chance, you can then collect all the outcomes and write them on the board to get a larger sample. This will make the students see the variability in the outcomes from one team to another. In Cycles Two and Three, it is important that the students realize the difference between an experiment involving 10 tries and one involving 100, so that they can identify which experiment's outcome is closer to the theoretical probability.

The meaning of the words in the subject-specific vocabulary of probability is also very important. Before beginning, ask the students what meanings they attach to the words they will have to use: random, chance, probable, possible, certain, impossible. For example, in general parlance, people speak of chance when no one knows what is going to happen, "It's up to chance!" But, when one refers to experiments connected with chance, one can attribute different degrees of certainty to possible outcomes and determine, from among these outcomes, those most likely to occur.

Jones et al. (1999) present six key concepts in probability: sample space, experimental probability of an event, theoretical probability of an event, probability comparisons, conditional probability and independence. The understanding of these concepts contributes to the development of probabilistic reasoning that the authors break down into four levels: Level 1, which is associated with subjective or non-quantitative reasoning; Level 2, which is considered transitional between subjective reasoning and naive quantitative reasoning; Level 3 which involves the use of informal quantitative reasoning; and Level 4, which incorporates numerical reasoning.

In accordance with the cycles

In Cycles One and Two, it is advantageous to begin with the extreme ends of the impossible-certain continuum by discussing examples of impossible events and certain events and then introducing possible events. Using a probability segment[3] to represent this impossible-certain continuum and trying to situate different events on this segment could make it easier to help students realize that some events are more or less likely to occur than others. The middle of the segment corresponds to an event that appears to be just as likely as unlikely. In Cycle Three, the same probability segment could be used to introduce the concept of probability as a decimal number (or fraction or percentage) between 0 and 1, starting with the meaning of a probability of 0 and of a probability of 1.

Younger children have a tendency to give the name of their favourite colour when they are asked what colour of marble is most likely to be drawn. The younger the child, the more important the emotional aspect is. Children may also attribute a memory to random phenomena. For example, if a child turns up "heads" four times in a row when tossing a coin, he or she may think that it is now time for "tails" to turn up. This naive reasoning is also seen in many adults who confess to choosing lottery numbers because "they haven't come up in a long time."

[3] Progression of Learning, p. 21, no. 5.

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In Cycle Three, in order to make the connection between experimental probability and theoretical probability, the same experiment can be simulated many times over. To do this within a reasonable length of time, using technology is recommended (see the links in the "Tools for teaching probability" section below). It is also useful to ask students to justify their answers when they evaluate probabilities (between 0 and 1) because a correct answer is not necessarily indicative of correct reasoning.[4]

Connections between probability and other branches of mathematics

It is useful to connect probability with other branches of mathematics and it is easy to do this with arithmetic and statistics.

In Cycle One, students count collections and, when they experiment with activities involving chance, they can use this skill to enumerate the possible outcomes of a simple random experiment or to identify equiprobability.

In Cycle Three, when probability must be expressed as a fraction (*part-whole* meaning), this application provides students with a new use for fractions. When the students have to situate various probabilities on a probability line, this gives them a new opportunity to situate fractions on a portion (between 0 and 1) of the numerical axis.

Furthermore, the problems set will allow students to use fractions, decimals or percentages to quantify a probability. They will also have to compare the outcomes of a random experiment with known theoretical probabilities, which will lead them to compare numbers with each other, whether they are expressed as fractions, decimals or percentages.

In all the cycles, performing experiments is recommended for integrating statistics and probability. Starting in Cycle One, students can use tables to collect, describe and organize data. In Cycle Three, students may be required to interpret circle graphs to make predictions. For example, using a graph that represents the favourite sports of the members of the class and that shows the percentages, you can ask the students to find the probability of randomly drawing the name of someone who likes one sport in particular. The students must find the number of people who like this sport by calculating the percentage it is of the total number of students and then represent the probability using a fraction, decimal or the percentage.

Tools for teaching probability

Manipulations in class are very important. For their in-class experiments, students can use various everyday objects: dice, marbles, cubes, prisms, glasses, playing cards, spinners, different coloured tokens or squares, two-coloured tokens, coins, thumbtacks.

In Cycles Two and Three, the use of technology is recommended for simulating random experiments with a large number of trials within a short period of time. Sites you can use include Scott Foreman Pearson's *E-tools* at www.pennsauken.net/~immath/etools/index.html or the virtual mathematics library at http://nlvm.usu.edu/en/nav/topic_t_5.html.

Learning and evaluation situations (LESs) on probability are available, in French only, on the Domaine MST Web site; for examples at the Cycle Two level, go to http://domaine.recitmst.qc.ca/Selon-le-hasard-des-jeux or http://domaine.recitmst.qc.ca/Une-histoire-improbable.

[4] Rubel, "Students' Probabilistic Thinking Revealed," 57.

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Bibliography and Webography

Association des promoteurs de l'avancement de la mathématique à l'élémentaire (APAME). "Les probabilités et statistiques au primaire… pourquoi pas?" *Instantanés mathématiques*, vol. XX, special issue B (1984).

Caron, France. "Probabilités et arithmétique: Une intersection." *Mathématique virtuelle à l'intention du primaire: Boîte à outils et idées pratiques* (2007). Accessed April 8, 2013. http://docmath.recitmst.qc.ca/public.php?service=files&dir=/math/files/doc_13.

E-tools. Pearson Scott Foresman. Accessed April 8, 2013. www.pennsauken.net/~immath/etools/index.html.

Jones, Graham A., Carol A. Thornton, Cynthia W. Langrall, and James E. Tarr. "Understanding Students' Probabilistic Reasoning." *Developing Mathematical Reasoning in Grades K-12. 1999 Yearbook.* Reston: National Council of Teachers of Mathematics, 1999.

MELS, Programme de formation de l'école québécoise - éducation primaire - enseignement préscolaire, Gouvernement du Québec, 2001.

MELS, Programme de formation de l'école québécoise, Progression des apprentissages au primaire, Mathématique, Gouvernement du Québec, 2009.

Poirier, Louise. *Enseigner les mathématiques au primaire*. Montréal: ERPI, 2001.

Rubel, Laurie H. "Students' Probabilistic Thinking Revealed: The Case of Coin Tosses." *Thinking and Reasoning with Data and Chance, 68th Yearbook.* Gail F. Burrill and Portia C. Elliot, editors. Reston: The National Council of Teachers of Mathematics, 2006.

Savard, Anne. "Le développement d'une pensée critique envers les jeux de hasard et d'argent par l'enseignement des probabilités à l'école primaire: Vers une prise de décision." Doctoral thesis. Québec: Université Laval, 2008. Accessed April 2nd, 2013. http://archimede.bibl.ulaval.ca/archimede/fichiers/25308/25308.pdf.

Selon le hasard des jeux: Primaire 2e cycle. Accessed April 2, 2013. http://domaine.recitmst.qc.ca/Selon-le-hasard-des-jeux.

Situation d'apprentissage: Une histoire improbable, 1re année du 2e cycle. Accessed April 8, 2013. http://domaine.recitmst.qc.ca/Une-histoire-improbable.

Utah State University. *Data Analysis & Probability.* Accessed April 8, 2013. http://nlvm.usu.edu/en/nav/topic_t_5.html.

Van de Walle, John A., and LouAnn H. Lovin. *Teaching Student-Centred Mathematics, Volume One: Grades K-3*. Toronto: Pearson Education Canada, 2007.

Van de Walle, John A., and LouAnn H. Lovin. *Teaching Student-Centred Mathematics, Volume Two: Grades 3-5*. Toronto: Pearson Education Canada, 2007.

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Excerpt from the Québec Education Program, p. 154

PROBABILITY

• Experimentation with activities involving chance	1	2	3
• Predicting the likelihood of an event (certainty, possibility or impossibility)	1	2	3
• Enumerating the possible outcomes of a simple random experiment	1		
• Probability that a simple event will occur (more likely, just as likely, less likely)		2	3
• Enumerating the possible outcomes of a random experiment using a table, a tree diagram		2	3
• Comparing the outcomes of a random experiment with known theoretical probabilities			3
• Doing simulations with or without a computer		2	3

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Excerpt from page 21 of the Progression of Learning in Elementary School: Mathematics, Québec Education Program

The table below presents the learning content associated with probability. The concepts and processes targeted will provide students with increasingly complex tools that will help them develop and use all three mathematics competencies.

→ Student constructs knowledge with teacher guidance. ★ Student applies knowledge by the end of the school year. Student reinvests knowledge.	Elementary					
	Cycle One		Cycle Two		Cycle Three	
	1	2	3	4	5	6
1. When applicable, recognizes variability in possible outcomes (uncertainty)	→	→	→	→	→	★
2. When applicable, recognizes equiprobability (e.g. quantity, symmetry of an object [cube])	→	→	→	→	→	★
3. When applicable, becomes aware of the independence of events in an experiment	→	→	→	→	→	★
4. Experiments with activities involving chance, using various objects (e.g. spinners, rectangular prisms, glasses, marbles, thumb tacks, 6-, 8- or 12-sided dice)	→	→	→	→	→	★
5. Predicts qualitatively an outcome or several events using a probability line, among other things						
a. certain, possible or impossible outcome	→	→	→	→	→	★
b. more likely, just as likely, less likely event			→	→	→	★
6. Distinguishes between prediction and outcome	→	→	→	→	→	★
7. Uses tables or diagrams to collect and display the outcomes of an experiment	→	→	→	→	→	★
8. Enumerates possible outcomes of						
a. a simple random experiment	→	★				
b. a random experiment, using a table, a tree diagram			→	→	→	★
9. Compares qualitatively the theoretical or experimental probability of events			→	→	→	★
10. Recognizes that a probability is always between 0 and 1					→	★
11. Uses fractions, decimals or percentages to quantify a probability					→	★
12. Compares the outcomes of a random experiment with known theoretical probabilities					→	★
13. Simulates random experiments with or without the use of technology			→	→	→	★
Vocabulary Chance, random experiment, enumeration, tree diagram Certain outcome, possible outcome, impossible outcome Event, likely, just as likely, more likely, less likely, event probability	→	→	→	★		

1. For example, if the pointer on a two-coloured spinner (red and yellow) stops on yellow three times, students will expect it to stop on red when it's their turn.

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