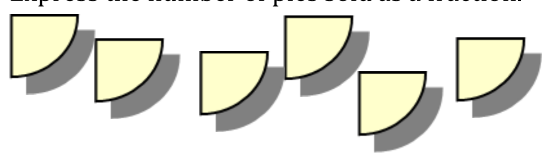
The same symbolization can have several meanings.

Take, for example, the fraction $\frac{3}{4}$.

The fraction $\frac{3}{4}$ can mean a whole divided into 4, from which 3 parts are taken. For example, we can say “three quarters of a cake, of a collection of objects, of a distance . . .” ***Part-whole* meaning** The whole is the reference point	The fraction $\frac{3}{4}$ can mean a ratio of 3 to 4. We can say: “Every time we count 4, we colour in 3.” ●●●○ | ●●●○ | Or: “For every 3 black marbles, there are 4 white ones.” ●●●○○○○ | ●●●○○○○ | and so forth ***Ratio* meaning** The whole is not always the reference point.	The fraction $\frac{3}{4}$ can mean 3 units (or 3 wholes) divided into 4. Thus, $\frac{3}{4}$ means $3 \div 4$. This case involves the *sharing* meaning of division. ***Indicated division* meaning** If several units (or wholes) are shared, the answer is nevertheless a fraction of a single unit (or whole). (If 3 pizzas are divided among 4 people, each person receives $\frac{3}{4}$ of **ONE** pizza.)
In the case of a set of 16 marbles, $\frac{3}{4}$ can refer to 3 parts of the set divided into 4 as per the *part-whole* meaning, but $\frac{3}{4}$ can also be seen as an operator to be applied to the set: $\frac{3}{4} \times 16$. **O*perator* meaning**	Moreover, $\frac{3}{4}$ can also mean three times $\frac{1}{4}$, where the quarter is considered a unit of measure. This concept is especially useful when a fraction seen as a p*art of a whole* is greater than 1. For example, $\frac{9}{4}$ cannot mean a whole divided into 4 from which 9 parts are taken, but $\frac{9}{4}$ can be better understood if the quarter is regarded as a unit of measure, or 9 times this unit. ***Measure* meaning**	

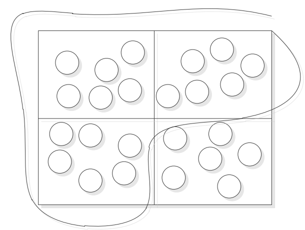
Some problems focus on one meaning rather than another.
Examples of problems involving the different meanings of fractions.

These beads represent one quarter of a bracelet. Find the total number of beads that make up the bracelet.  ***Part-whole* meaning** This problem focuses on the *part-whole* meaning, since students must consider the whole in their reasoning.	Mary always uses the same ratio of black beads and white beads to make bracelets and necklaces. This ratio is 3 to 4 or $\frac{3}{4}$. Draw three different bracelets that Mary can make. ***Ratio meaning*** This problem focuses on the *ratio* meaning. Students can make the bracelets as long as they want, and they do not have to refer to the whole.	Four children share 3 pizzas. What fraction of a pizza will each child receive? $3 \div 4$ ***Indicated division* meaning** This type of problem also shows that $3 \div 4 = \frac{3}{4}$, thereby demonstrating that there are different ways of writing the same mathematical idea.
Secondary school problems that involve enlarging or reducing objects (dilatation) are good examples of how the *operator* meaning of a fraction comes into play. Mary wants to give her brother $\frac{3}{4}$ of her set of marbles. If Mary has 16 marbles, how many marbles will she give her brother? ***Operator* meaning** This type of problem can develop the student's understanding of the ***operator*** meaning of the fraction. It involves multiplying a natural number by a fraction: $16 \times \frac{3}{4}$. However, this problem will probably be solved by using the *part-whole* meaning: $\frac{3}{4}$ of 16.	Mary sells pies in slices. Each slice represents $\frac{1}{4}$ of each pie. Mary sold all her slices of pie. Express the number of pies sold as a fraction.  ***Measure* meaning** The fraction $\frac{1}{4}$ is a unit of measure used to determine the number of pies sold. There are 6 slices each representing $\frac{1}{4}$ of a pie for a total of $\frac{6}{4}$.	

Some interpretations of the *part-whole* and *ratio* meanings of fractions

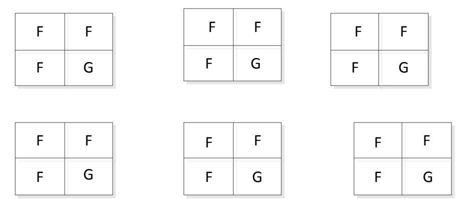
If a unit (or a fixed whole) is given, using a *part-whole* fraction model or a *ratio* model to represent a fraction will yield the same result.
For example: Ms. Miller has broken her class up into small work groups. Each group consists of 4 people, $\frac{3}{4}$ of whom are girls. How many girls are in this class of 24 students?

Fraction Model



There are 18 girls
in Ms. Miller's class.

Ratio Model



There are 18 girls
in Ms. Miller's class.

If students must illustrate a fraction without a fixed whole, they can proportionally increase the whole as much as they want. In this case, the different illustrations represent equivalent ratios, i.e. the surfaces occupied are proportional, but the quantities are not equal. The focus is on the *ratio* meaning in proportional situations where the number of total parts is not equal to the denominator.

For example: In art class, Lisa made a series of drawings consisting of squares and octagons. She always used the same ratio of squares and octagons, and $\frac{3}{4}$ of the figures were squares. Make 3 different sketches that Lisa could have drawn.