Cultural, Social and Technical Option in Secondary V

APPENDIX: EXAMPLES OF FINANCIAL MATHEMATICS PROBLEMS

Excerpts from the Québec Education Program (QEP), Chapter 6, p. 68

Understanding real numbers, algebraic expressions and dependency relationships (cont.)	
– Financial mathematics • Simple and compound interest • Interest period • Discounting (current value) • Compounding (future value)	– Analyzing situations related to economics (e.g. personal finances), social issues, technical or scientific contexts, or everyday life • Switching from exponential to logarithmic notation and vice versa • Solving exponential and logarithmic equations using a change of base, if necessary – Calculating, interpreting and analyzing financial situations

Note: In Secondary IV, students use a graph, a table of values or technology in situations in which they must determine the value of the exponent.

Students differentiate, recognize and analyze various families of real functions and are presented with situations involving real functions expressed as follows: quadratic functions of the form $f(x) = ax^2$ and exponential functions of the form $f(x) = ab^x$ where $a \neq 0$ and $b > 0$. For the other functions, students may be given rules for which they are able to calculate values, sketch graphs and analyze properties, but without having to represent the situation algebraically.

In Secondary V, students use a graph, a table of values or a calculator in situations in which they must determine the approximate value of the exponent (logarithm). To determine this value, they manipulate expressions and convert them to the same base (base 10, for the calculator) so as to make the exponents comparable.

If necessary, they use equivalences such as $a^b = c \leftrightarrow log_a c = b$ **or** $log_a c = \frac{log_b c}{log_b a}$.

In the case of situations involving personal finances, different aspects may be taken into account:
– types of income, such as types of compensation, salaries, commissions, contracts and gratuities
– different types of taxation, such as income tax, property tax and deductions at source
– types of financing, such as purchase options, personal loans, mortgages and financing costs
– the cost of certain utilities, such as the telephone or electricity

When assessing financial situations in Secondary V, students use rules such as $C_n = C_0(1 + i)^n$ to determine compounding and $C_0 = C_n(1 + i)^{-n}$ **or** $C_0 = \frac{C_n}{(1+i)^n}$ to determine discounting (where Cn = future value, C0 =current value, i = interest rate and n= interest period).

Although this is implied by the rules presented in the program, it is important to specify that they involve <u>interest paid out once per period</u>, such as annual interest (paid out or compounded once a year) or monthly interest (paid out or compounded once a month). The idea is to avoid using the formula $C_n = C_0\left(1 + \frac{i}{t}\right)^{nt}$. However, this formula could be one of the avenues of exploration to be pursued with the students.

Examples of financial problems that students could be asked to solve:

1. \$50 000 is invested for 5 years at an interest rate of 6% compounded every year.

Value of the investment after 5 years:

n = 5

$$C_n = C_0(1+i)^n = 50\,000(1+0.06)^5$$
$$= 50\,000(1.06)^5$$
$$= \$66\,911.28$$

2. \$50 000 is invested for 3 months at an annual compound interest rate of 6%.

Value of the investment after 3 months:

n = 0.25 (since 3 months correspond to a quarter [0.25] of a year)

$$C_n = C_0(1+i)^n = 50\,000(1+0.06)^{0.25}$$
$$= 50\,000(1.06)^{0.25}$$
$$= \$50\,733.69$$

3. \$50 000 is invested for 3 months at a monthly compound rate of 0.6%.

Value of the investment after 3 months:

n = 3

$$C_n = C_0(1+i)^n = 50\,000(1+0.006)^3$$
$$= 50\,000(1.006)^3$$
$$= \$50\,905.41$$

4. \$5 000 is invested for 10 years at a monthly compound interest rate of 0.1%.

Value of the investment after 10 years:

n = 120 (since the interest is paid out every month, we must calculate how many months there are in 10 years: there are 12 months per year and therefore 120 months in 10 years)

$$C_n = C_0(1+i)^n = 5\,000(1+0.001)^{120}$$
$$= 5\,000(1.001)^{120}$$
$$= \$5\,637.15$$

5. \$5 000 is invested for 10 years at an interest rate of 1% compounded every 3 months.

Value of the investment after 10 years:

n = 40 (since the interest is paid out every 3 months, we must calculate how many 3-month periods there are in 10 years: there are four 3-month periods per year and therefore 40 periods in 10 years)

$$C_n = C_0(1+i)^n = 5\,000(1+0.01)^{40}$$
$$= 5\,000(1.01)^{40}$$
$$= \$7\,444.32$$

6. \$5 000 is invested for 10 years at an interest rate of 6% compounded every 2 years.

Value of the investment after 10 years:

n = 5 (since the interest is paid out every 2 years, we must calculate how many 2-year periods there are in 10 years: there are five 2-year periods)

$$C_n = C_0(1+i)^n = 5\,000(1+0.06)^5$$
$$= 5\,000(1.06)^5$$
$$= \$6\,691.13$$

The formula $C_n = C_0(1+i)^n$ applies only if the interest rate and the term of the investment are expressed in the same unit of time as the compounding period. For example, if a financial institution and an investor agree that the interest is compounded at the end of each month, the formula will apply only if the interest rate is monthly and the term of the investment is expressed in months.