Understanding proportionality

The development of proportional reasoning is essential, and it has many applications both within and outside mathematics. For example, students use percentages (calculating a certain percentage of a number and the value corresponding to 100 per cent) in situations relating to consumption, probability and statistics. In working with graphs for example, they make scale drawings and construct circle graphs. They look for unknown values in algebraic or geometric situations (e.g. similarity transformations, arc lengths, sector areas, unit conversion).

An understanding of proportions can be developed when students interpret ratios or rates in various situations, compare them qualitatively or quantitatively (e.g. “a is darker than b,” “c is less concentrated than d”) and describe the effect of changing a term, a ratio or a rate.

Procedures students may use to deal with a proportionality situation

Once students are able to recognize a proportional situation, they can express it as a proportion. They then solve it by using multiplicative strategies that they will have developed (e.g. unit-rate method, factor of change, ratio or proportionality coefficient, additive or mixed procedure). A minimum of three ordered pairs is required to analyze a proportional situation using a table of values.

Examples :

Quantity of product A	2	4	6	10
Quantity of product B	6	12	18	?

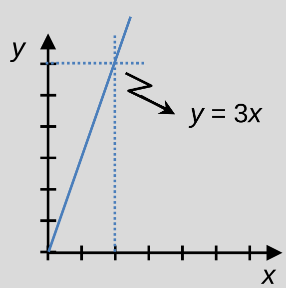
Unit rate method : If for 1 unit of product A, we have 3 units of product B (12 ÷ 4), then for 10 units of product A, we will have (10 □ □ 3) units of product B.

Factor of change : The factor that makes it possible for 4 to be increased to 10 is 2.5; we apply this factor to 12.

Proportionality coefficient : The factor that makes it possible for 4 to be increased to 12 is 3; we apply this factor to 10.

Additive procedure : Since 4:12 = 6:18, then $\frac{4}{12}=\frac{6}{18}=\frac{4+6}{12+18}=\frac{10}{30}$

Graphic procedure :
(2 Secondary Cycle Two)



x	2	3	5	…	16
$y = f(x) = 3x$	6	9	15	…	48

$f(x + y) = f(x) + f(y)$ $\quad$ $f(2 + 3) = f(2) + f(3)$

$f(kx) = k\,f(x)$ $\quad$ $f(8 \text{ x } 2) = 8 \text{ x } f(2)$

$f(ax + by) = af(x) + bf(y)$ $\quad$ $f(2(3) + 3(5)) = 2f(3) + 3f(5)$

Types of proportionality

In order to fully understand the concept of proportionality, the students must work with a wide range of situations in various contexts. The table below lists several types of proportionality to consider when drawing up lesson plans.

<table>
<tr><th>Type of proportionality</th><th colspan="5">Situation</th><th>Model</th></tr>
<tr><td rowspan="6">Simple and direct
"This involves two quantities with only two magnitudes considered for each quantity. In all, three values and one unknown are usually involved."[1]</td><td rowspan="5">Fourth Proportional Situation</td><td rowspan="4">One of the numbers is equal to 1.</td><td colspan="2">Multiplication situation</td><td>1 kg of apples costs $1.20. How much does 5 kg of apples cost?</td><td>× 5
Mass (kg) | 1 | 5
Cost ($) | 1,2 | ?
× 5</td></tr>
<tr><td rowspan="3">Division situation</td><td>Sharing</td><td>We paid $10.80 for 12 AAA batteries. How much does 1 battery cost?</td><td>÷ 12
Number of batteries | 12 | 1
Price($) | 10,80 | ?
÷ 12</td></tr>
<tr><td rowspan="2">Number of times x goes into y</td><td>At the drugstore, the pharmacist dispensed 390 pills into bottles that hold 30 pills. How many bottles did he fill?</td><td>Number of pills | 30 | 390
Number of bottles | 1 | ?
÷ 30</td></tr>
<tr><td>A train travelled 476 km at an average speed of 140 km/h. How much time did it take the train to cover this distance?</td><td>Distance travelled (km) | 140 | 476
Time (h) | 1 | ?
÷ 140</td></tr>
<tr><td>None of the numbers is equal to 1.</td><td colspan="2"></td><td>Jane paid $28 for 3 photo albums.
How much would 5 albums cost?</td><td>Number of albums | 3 | 5
Price ($) | 28 | ?</td></tr>
<tr><td colspan="5">Comparison situation
Colour A is created by adding 11.5 drops of red paint to 3 litres of white paint.
Creating colour B involves adding 26 drops of red paint to 7 litres of white paint.
Which colour is pinker, A or B?</td><td>They must compare the relationships (<, =, >) :
$\frac{11{,}5}{3} \; ? \; \frac{26}{7}$
or 11,5 : 3 ? 26 : 7</td></tr>
</table>

[1] 1. This typology is based on Vergnaud's theory, which was used by Christine Géron, Pierre Stegen and Sabine Daro in *L'enseignement de la proportionnalité*, Chapter 1, 2007, pp. 28, 30 and 32. [Online] 2010 [www.enseignement.be/index.php ?page=2382&do_id=2712&do_check=].

<table>
<tr>
<td>Simple compound
“This involves more than two quantities. It can be solved by calculating a simple proportion for two pairs of quantities in succession.”</td>
<td>An office supply store sells a company 5 packs of scissors. There are 6 pairs of scissors in each pack. Given that a pair of scissors costs $19.99, how much should the company pay for all the scissors?</td>
<td>
<table>
<tr><td>Number of packs</td><td>5</td><td>1</td><td></td></tr>
<tr><td>Number of pairs of scissors</td><td></td><td>6</td><td>1</td></tr>
<tr><td>Price ($)</td><td>?</td><td></td><td>19,99</td></tr>
</table>
<table>
<tr><td>Number of packs</td><td>1</td><td>5</td></tr>
<tr><td>Number of pairs of scissors</td><td>6</td><td>?</td></tr>
</table>
30
× 5
<table>
<tr><td>Number of pairs of scissors</td><td>1</td><td>30</td></tr>
<tr><td>Price ($)</td><td>19,99</td><td>?</td></tr>
</table>
× 30
</td>
</tr>
<tr>
<td>Double or multiple
“This consists of more than two quantities, but cannot be broken down into several successive problems involving simple proportionalities. To find the unknown value, students must simultaneously associate several unrelated pieces of information.”</td>
<td>A car rally in the desert lasts 28 days and involves 40 participants. Typically, in the desert, you need 350 litres of water per week for ten people. How much water is needed for this rally?</td>
<td>
<table>
<tr><td>Number of weeks</td><td></td><td>1</td></tr>
<tr><td>Number of days</td><td>1</td><td>7</td></tr>
<tr><td>Number of people</td><td>10</td><td>x</td></tr>
</table>
70
× 7
<table>
<tr><th>Number of days</th><th>Number of people</th><th>Amount of water (l)</th></tr>
<tr><td></td><td>x = 70</td><td>350</td></tr>
<tr><td>1</td><td>1</td><td>y = 5</td></tr>
<tr><td>1</td><td>40</td><td>z = 200</td></tr>
<tr><td>28</td><td></td><td>?</td></tr>
</table>
5 600
</td>
</tr>
</table>

Proportionality table: a useful tool . . .

By dividing by 3, we get the unit rate…

$+$ $\quad$ $\times 4$

Number of flasks	3	5	6	8	24
Number of litres	1,2	2	2,4	3,2	9,6

$\div 0{,}4$ $\quad$ $\times 0{,}4$

$+$ $\quad$ $\div 4$

By dividing by 3, we get what corresponds to the unit rate…