TYPES OF REASONING TO BE USED IN EXERCISING THE COMPETENCY : USES MATHEMATICAL REASONING

In exercising the competency *Uses mathematical reasoning*, students must make conjectures and draw up proofs. They are required to use different types of reasoning to clarify, validate, adjust or refute conjectures made by themselves or others. However, it is important that they learn to distinguish between reasoning and a mathematical proof. They use their reasoning when they examine a situation, determine how they will deal with it and organize their thinking. They prove a conjecture when they give an explanation that is "assumed to be true by a given community (e.g. the class, the general population, the mathematics community, the scientific community) and takes into account the audience and the degree of rigour required in an argument."[1] They show that a statement is true when its proof, assumed to be true by the scientific community, respects certain rules and the mathematical objects on which it is based have a theoretical status.[2] Furthermore, "critically assessing, refuting or justifying a statement and correctly expressing one's ideas are skills that can be applied well beyond the field of mathematics and reflect an individual's general education. Exploring a wide variety of situational problems throughout one's schooling is designed to develop this ability."[3]

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In developing the competency to use mathematical reasoning, in Secondary Cycle One, students have already been introduced to a few basic rules of deductive reasoning and have learned to distinguish between properties validated experimentally and those established by deduction. They have also learned to justify the steps in their reasoning by using statements and definitions or by finding counter-examples.

In each branch of mathematics, students must use specific types of reasoning. For example, statistical reasoning involves interpreting a set of results or a graph as well as the concepts of probability and statistical data, whereas algebraic reasoning calls on a sense of numbers or operations. However, "any reasoning uses the process of inference (if . . . then . . .) as a foundation, i.e. starting with a premise and

[1] Québec Education Program, 26

[2] Gilbert ARSAC et al., *Initiation au raisonnement déductif au collège. Une suite de situations permettant l'appropriation des règles du débat mathématique*, Lyon, Presses universitaires de Lyon, 1992.

[3] Réciproques, Pédagogie, Équipe académique Mathématiques, No. 15, May 2001. [Online] [http://mathematiques.ac-bordeaux.fr/profplus/publica/bulletin/bull15/raison.htm].

ending with a conclusion. Errors in reasoning are due to false or inconsistent premises, since if we can deduce what is true and what is false, the inverse is not possible."[4]

In addition to the types of reasoning specific to each branch of mathematics, students can use more general types of reasoning, including:

- **DEDUCTIVE REASONING**

Process by which students reach a conclusion based on principles considered to be true. When using this type of reasoning, students go from the general to the specific. Each step in the reasoning consists of a "deductive island" that includes an initial proposition, a rule of inference (definitions, axioms, theorems, etc.) and a final proposition which becomes the initial proposition of another step, as applicable. this type of reasoning also includes proof by exhaustion and proof by contradiction.

"Geometry is an ideal tool for exercising deductive reasoning. But geometric reasoning is also based on the observation and construction of figures, experimentation, trial-and-error procedures, and the development and critical assessment of conjectures. Geometry is a rich and varied field of mathematical reasoning, which has a visual, esthetic and even playful dimension and involves different types of reasoning. In order for geometric reasoning to be learned effectively, it must not be limited to the formal learning of proofs. To this end, statements should not be systematically presented in a closed manner: "show that" followed by a property that is as self-evident to the students as hypotheses. Students then find that geometric activity is an incomprehensible and sterile game."[5]

- **INDUCTIVE REASONING**

Process by which students identify rules or principles and generalize on the basis of specific cases. Systematic or guided trial strategies are associated with this type of reasoning.

"Generalization through inductive reasoning guarantees neither certainty nor truth. For example, given the result of a poll of a sample of a population, we can infer that the result would be fairly similar for the population as a whole. This generalization can obviously not be made with any certainty."[6] "In mathematics, inductive reasoning is usually regarded as an initial step leading to a conjecture. Deductive reasoning must then be used to demonstrate the truth of this conjecture."[7]

[4] Classification of types of reasoning [Online] [http://projetconnaissance.free.fr/classement/classements/classement_raisonnements.html].
[5] Ressources pour les classes de 6e, 5e, 4e, et 3e du collège, Raisonnement et démonstration [Online] [eduscol.education.fr/ D0015].
[6] Sciences de l'ingénieur, le plaisir du raisonnement [Online] [http://www.si.ens-cachan.fr/accueil_V2.php?id=24&page=affiche_ressource].
[7] Ressources pour les classes de 6e, 5e, 4e, et 3e du collège, Raisonnement et démonstration [Online] [eduscol.education.fr/ D0015].

- **ANALOGICAL REASONING**

Process by which students make comparisons based on similarities in order to make conjectures or draw conclusions. An awareness of how situations are structured makes this type of reasoning possible.

- **PROOF BY EXHAUSTION**

Process by which students validate a conjecture by making sure it covers all possible cases. Unlike an exhaustive study, in proof by exhaustion situations are differentiated on the basis of one or more parameters.

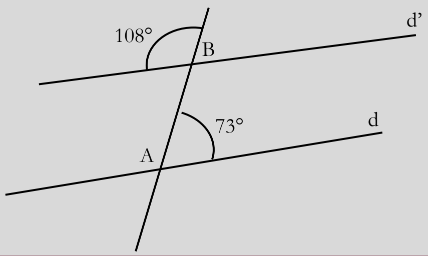
- **PROOF BY CONTRADICTION**

Process by which students validate a conjecture by showing its negation leads to a contradiction. This type of reasoning is based on the fact that the "not not-true" can only be true.

- **REFUTATION USING COUNTEREXAMPLES**

Process by which students refute a conjecture by evoking a case or example for which the conjecture is false. Students must be made to understand that producing numerous examples is not a proof, whereas a single counter-example is a proof.

A FEW EXAMPLES

Deductive reasoning	• What is the graph of the function $f(x) = 2x^2 + 3x + 4$? • Is the expression cosec A(cosec A – sin A) = $\cot^2$ A a trigonometric identity?
Inductive and analogical reasoning and refutation using counterexamples	• If the dimensions of a rectangle are doubled, then its perimeter is doubled. If the dimensions of a rectangle are tripled, then its perimeter is tripled. This statement is generalized. Does this apply to areas?
Proof by exhaustion	• Let a, b and c be three relative whole numbers. Let $abc = 32$, $a < b < c$ and $bc < 0$ All possible solutions are found. • What is the last digit in 2^{50}?
Proof by contradiction	• Are lines l and l' parallel? 

In exercising the competency *Uses mathematical reasoning*, students should not be asked to carry out specific tasks by using a specific type of reasoning; rather they should be able to use different types of reasoning in the situations.

For example,

Is $3^n + 1$ even for all n integers?

Proof by exhaustion	Consider the units digit of $3^n + 1$. The possible units digits for the powers of 3 are 1, 3, 7 or 9. By adding 1, the possible units digits for $3^n + 1$ are therefore 2, 4, 8 or 0. Thus, $3^n + 1$ is even.
Proof by contradiction	If $3^n + 1$ is odd, then it is of the form $2k + 1$, where k is a natural integer. Since $3^n = 2k$, 2 will appear as a factor in the prime factorization of 3^n. However, 3 is the only factor in the prime factorization of 3^n. There is a contradiction, therefore $3^n + 1$ is even.