A Collection of
Questions and Answers

Elementary School

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General questions about the Québec Education Program (QEP) and the Progression of Learning

1. Is arithmetic more important than the other branches of mathematics?

According to the QEP, all of the subject-specific competencies, as well as their key features, **must** be developed through **all the essential knowledge**.

The Progression of Learning says that "the concepts and processes to be acquired and mastered in arithmetic constitute the building blocks of mathematics, since they are applied in all other branches of this subject.*"* This statement qualifies the importance of arithmetic, treating it as a **basic element** of mathematics without diminishing the importance of mastering the other concepts and processes related to the other branches of mathematics.

References: QEP, p. 9
Progression of Learning, p. 4

2. Does the Progression of learning take precedence over the QEP?

The Progression of Learning:

- ✓ complements the QEP
- ✓ includes a **breakdown of the essential knowledge** in the QEP
- ✓ provides **specifications regarding the knowledge and skills** students must acquire **each year of elementary school** in each branch of mathematics
- ✓ shows how and when the compulsory knowledge and skills are to be acquired **over the six years of elementary school**
- ✓ specifies **actions** to be performed in order for students to fully assimilate the knowledge and skills
- ✓ contains tables **showing how student learning progresses**
- ✓ illustrates the mathematical symbols and vocabulary
- ✓ contains examples of strategies
- ✓ helps teachers with their lesson planning

The *Progression of Learning* document does not replace the QEP, but complements it.

Understanding and writing numbers

Natural numbers

❖ Representing numbers: grouping and place-value tables

In grade 1, should we use place-value tables to represent numbers in different ways or to associate a number with a group of objects or group of pictures?

Using place-value tables in grade 1 is premature. Instead, emphasis should be placed on **apparent, accessible groupings** using objects, drawings or **unstructured materials** such as tokens, nesting cubes, groups of ten objects placed inside a bag and ten of these bags placed inside another container.

It is **only in grade 2** that emphasis is placed on **exchanging apparent, non-accessible groupings, using structured materials** such as base ten blocks and place-value tables. In cycles two and three, place-value tables are also useful for working with **place value**.

References: QEP, p. 134
Progression of Learning, pp. 5-6, no. A-4. a-b-c

❖ Place-value tables

What is the purpose of a place-value table?

Place-value tables are used when working with **exchanges** and **place value** to represent numbers.

References: QEP, p. 150
Progression of Learning, pp. 5-6, no. A-4. a-b-c

❖ Counting in base ten

When should we introduce the following type of reasoning: "How many tens are in the number 234?" (Ans.: 23)

This question involves the development of students' number sense and, more specifically, learning to count in base ten. The concept of counting in base ten is constructed gradually. The first six statements in the section *Understanding and writing numbers* (Progression of Learning, pp. 5-6) and the corresponding essential knowledge (QEP, p. 150) describe the first steps in developing number sense.

Students should be asked to carry out various activities. They will learn to count and count collections of actual or drawn objects. Students count collections by counting each object.

They then group and associate these groups with numbers. Eventually, they will learn to do exchanges. Concurrently, they learn to read and write numbers and are introduced to the related vocabulary: unit, tens place, hundreds place. They will be encouraged to represent natural numbers in different ways, to associate a number with a set of objects or to draw and recognize equivalent representations of numbers. Various manipulatives, such as tokens, objects, nesting cubes, base ten blocks and place-value tables, should be made available to students. Putting emphasis on place value enables students to evolve from the concrete to the symbolic.

For example: students will count 23 objects by counting them one by one. Then, they will learn how to form other groups (including groups of 10) and how to count this collection by grouping (e.g. they could count 10, 11, 12, . . . 22, 23 or count 10, 20, 21, 22, 23). They could write 23 and say 23 or 1 ten and 13 units or 2 tens and 3 units. **In doing these types of activities students will come to realize that the number 23 is made up of 2 tens and 3 units**, and they will be able to represent this collection in different ways and recognize equivalent representations.

According to statement 4 on pages 5 and 6 of the Progression of Learning, it would seem premature to engage in this type of activity before Cycle Two.

References: QEP, p. 150
Progression of Learning, pp. 5-6, nos. A-1, A-2, A-3, A-4, A-5 and A-6

- **Decomposing numbers**

When elementary school students are required to decompose numbers in different ways, can we go so far as to have them use exponents?
e.g.: $84\,123 = 8 \times 10^4 + 4 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 + 3 \times 1$

In elementary school, students are not expected to use exponents in decomposing numbers. They may, for example, decompose the number 84 123 as follows:

84 123 = 80 000 + 4 000 + 100 + 20 + 3
84 123 = 80 000 + 4 100 + 20 + 3
84 123 = 8 X 10 000 + 4 X 1 000 + 1 X 100 + 2 X 10 + 3 X 1
84 123 = 84 X 1 000 + 12 X 10 + 3 X 1
84 123 = 8 ten thousands + 4 thousands + 1 hundred + 2 tens + 3 units
84 123 = 84 thousands + 12 tens + 3 units
84 123 = 4 X 21 000 + 3 X 40 + 3
84 123 = 2 X 42 000 + 130 – 7

References : QEP, p. 150
Progression of Learning, p. 6, no. A-5

As far as exponents are concerned, Cycle Three students are expected to **calculate the power of a number.**

e.g.: Calculate 6^3 $6 \times 6 \times 6$ Answer: 216

e.g.: There are five houses on Feline Street. There are five cats in each house. Each cat caught five mice. Each mouse had eaten five peanuts.

Using exponents, write the total number of:

- cats 5×5 Answer: 5^2
- mice $5 \times 5 \times 5$ Answer: 5^3
- peanuts $5 \times 5 \times 5 \times 5$ Answer: 5^4

References: QEP, p. 150
Progression of Learning, p. 12, no. A-10

Understanding and writing numbers

Fractions

❖ Fractional notation/mixed numbers

Does fractional notation include mixed numbers?

A mixed number is an **integer followed by a fraction** (e.g.: $2\frac{1}{2}$, $7\frac{5}{6}$), whereas fractional notation is the **representation of a number** as a quotient of two numbers ($\frac{a}{b}$). For example, $3\frac{1}{2}$ is a mixed number expressed in fractional notation, and 3.5 is its decimal notation. Mixed numbers are covered in elementary school.

References: QEP, p. 155 (Symbols)
Progression of learning, p. 7, (Vocabulary, Cycle Two)

Fractional notation

Representation of a number, a numerical expression or an algebraic expression in the form of a quotient of two numbers, numerical or algebraic expressions.

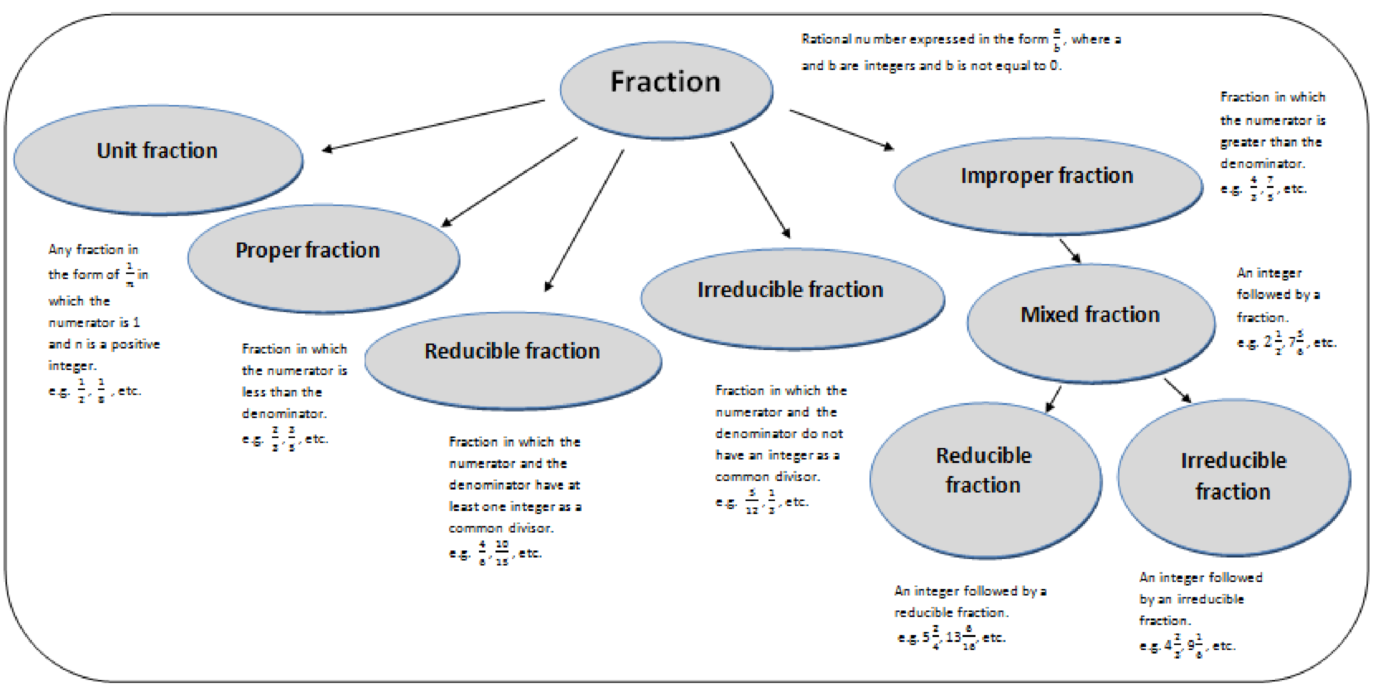
e.g. $\frac{15}{4}$, $\frac{5}{6}$, $2\frac{1}{3}$, etc.

Decimal notation

Representation of a real number with one part being a whole number and the other a decimal: the two parts are separated by the decimal point.

e.g. 12.75, 1.333..., etc.

Source: De Champlain, Denis et al. *Lexique mathématique – enseignement secondaire.* 2nd ed., revised and corrected. Montréal: Les Éditions du Triangle d'Or, 1996.



Source: De Champlain et al., *Lexique mathématique – enseignement secondaire*, 2d ed. revised and corrected. Montréal: Édition du Triangle d'Or, 1996.

Decimals

❖ Decimal notation

How do you write the number 32? Is it acceptable to write 32.00?

The number 32 can be written in decimal form. It is not wrong, but it is not always useful **in certain contexts**. It is relevant when we want to compare or order numbers and to perform operations and when it is required by the context, for example when referring to money. Writing it in decimal form out of context may raise questions: Why stop at 2 zeros after the decimal point, why not use 3?

Typically, decimal notation indicates that the unit place value is immediately to the left of the decimal point. When there is no decimal component in the quantity involved, decimal notation is not useful.

References: QEP, p. 150
Progression of Learning, p. 7, nos. C-3 and C-4

❖ Composition and decomposition

What are the different possibilities Elementary Cycle Two students could use to compose and decompose a decimal written in decimal notation?

Can we talk about fractions, parentheses, place value tables, . . . ?

Let's look at the limits regarding the size of the numbers that Cycle Two students must compose and decompose:

- the **decimal** part of the number should not go beyond the second decimal place
- the **integer** part of the number is limited to 100 000

The section ***Understanding and writing numbers – Natural numbers*** in this document provides several examples of models of decomposition for the integer part of the number to be decomposed.
For the decimal part of the number to be decomposed, it is important that students use decimal notation. Here are some examples:
$6.31 = 6 + 0.3 + 0.01$
$5.46 = 5 \text{ X } 1 + 46 \text{ X } 0.01$
$1.5 = 15 \text{ X } 0.1$ because the student recognizes equivalent expressions
At the same time, students also use fractional notation when decomposing the numbers. For example:
$6.31 = 6 + \frac{3}{10} + \frac{1}{100}$ because the student associates the decimal with a fraction.

In a decomposition such as $12.3 = (1 \times 10) + (2 \times 1) + (3 \times \frac{1}{10})$, it is not necessary to use the parentheses, but they help in visualizing the place value of the digits in the number being decomposed. In the same way, a place value table may also be helpful in showing the place value of the digits in a decimal that is to be decomposed.

References: QEP, p. 150
Progression of Learning, p. 7 nos. C-5, C-6 and B-9

Meaning of operations involving numbers

Natural numbers

❖ Combination of transformations

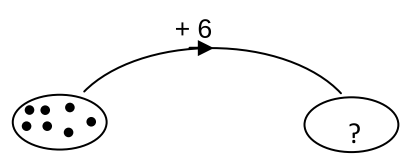
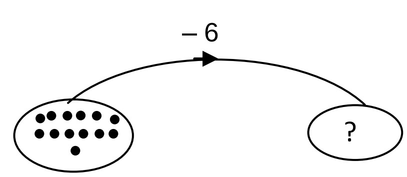
*What does **combination of transformations** mean on page 9 of the Progression of Learning? What does this concept of addition and subtraction mean?*

The following is a table of *Additive structures* summarizing the different meanings of addition and subtraction covered in elementary school. It includes combinations of transformations.

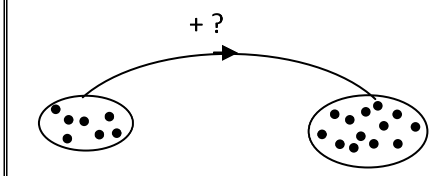
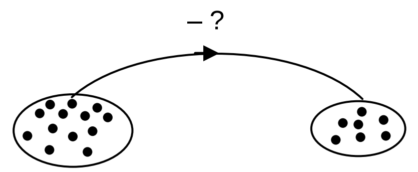
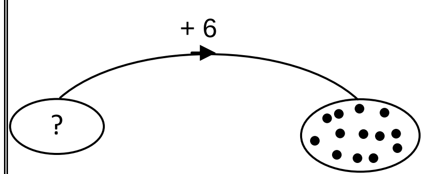
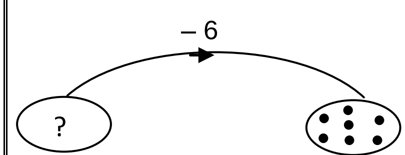
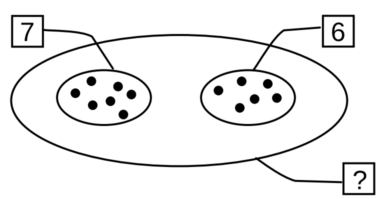
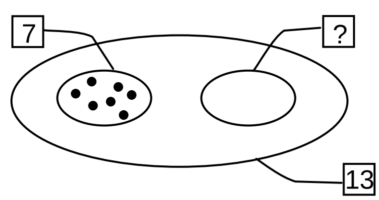
References: QEP, p. 151
Progression of Learning, p. 9, no. A-2 b and c

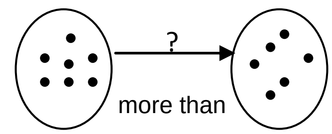
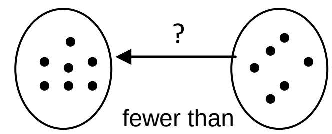
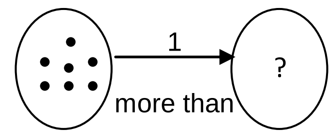
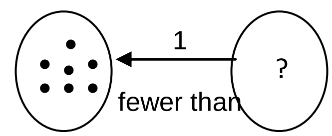
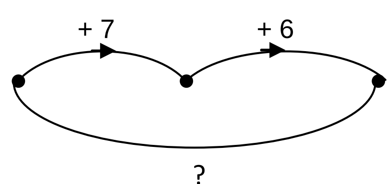
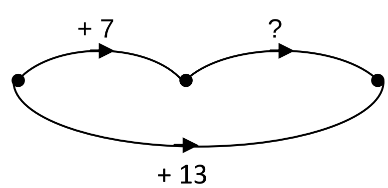
Additive structures

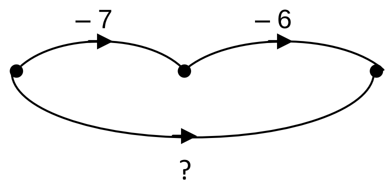
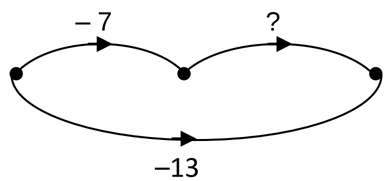
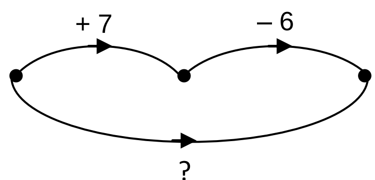
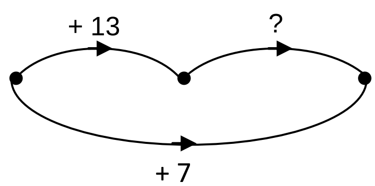
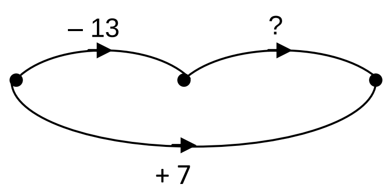
Techniques for performing operations, relationships between operations and the properties of operations only have real meaning when they are used to mathematize situations in order to solve problems. *Additive structures* deal with addition and subtraction, regardless of the type of numbers involved. It is essential that a variety of situations be presented in each cycle of elementary school: change (adding or taking away), uniting, comparing (more or fewer than) and combination of changes (positive, negative or mixed). It is not necessary for students to know or memorize the different names for these structures. They must instead develop their own ways of representing them. The table below presents a variety of situations involving varying levels of difficulty.

Structure or meaning	**Situation**[1]	**Model (Students create their own representations according to the situation)**	**Equation**
Transformation (adding) Determine the final state	Gus had 7 objects. Melanie gave him 6 more. How many objects does Gus have now?		7 + 6 = ❒
Transformation (taking away) Determine the final state	Gus had 13 objects. He gave 6 of them to Melanie. How many objects does Gus have now?		13 – 6 = ❒

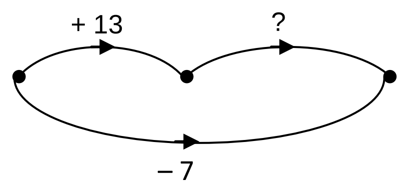
[1] The following examples consist of only two values each. Teachers should make sure to present situations consisting of several values and involving more than one meaning as well as superfluous or missing data.

Transformation (adding) Determine the change involved	Gus had 7 objects. Melanie gave him some objects. Gus now has 13 objects. How many objects did Melanie give Gus?		7 + ❐ = 13
Transformation (taking away) Determine the change involved	Gus had 13 objects. He gave some to Melanie. Gus now has 7 objects. How many objects did Gus give Melanie?		13 – ❐ = 7
Transformation (adding) Determine the initial state	Gus had some objects. Melanie gave him 6 more. Gus now has 13 objects. How many objects did Gus start with?		❐ + 6 = 13
Transformation (taking away) Determine the initial state	Gus had a certain number of objects. He gave 6 to Melanie. He now has 7 objects. How many objects did Gus start with?		❐ – 6 = 7
Uniting Determine the set	Gus has 7 objects. Melanie has 6. How many objects do they have in all?		7 + 6 = ❐
Uniting Determine a subset (complement)	Melanie and Gus have 13 objects all together. Gus has 7. How many objects does Melanie have?		7 + ❐ = 13 13 – 7 = ❐

Comparing (“more than”) Determine the comparison	Gus has 7 objects. Melanie has 6. How many more objects than Melanie does Gus have?		7 = 6 + ❒ 7 – ❒ = 6
Comparing (“fewer than”) Determine the comparison	Gus has 7 objects. Melanie has 6. How many fewer objects than Gus does Melanie have?		7 = 6 + ❒ 7 – ❒ = 6
Comparing (“more than”) Determine a set	Gus has 7 objects. He has 1 more object than Melanie. How many objects does Melanie have?		7 – 1 = ❒ 7 = ❒ + 1
Comparing (“fewer than”) Determine a set	Gus has 7 objects. Melanie has 1 fewer object than Gus. How many objects does Melanie have?		7 – 1 = ❒ 7 = ❒ + 1
Combination of transformations (positive) Determine the gain	Yesterday, Gus received 7 objects. Today, he has received 6 more. How many objects has he received over the 2 days?		7 + 6 = ❒
Combination of transformations (positive) Determine the change involved	Yesterday, Gus received 7 objects. Today, he has received more but we don’t know how many. Given that he has received 13 objects in the past 2 days, has he received more or fewer objects today than he did yesterday? How many objects has he received today?		7 + ❒ = 13

Combination of transformations (negative) Determine the loss	Yesterday, Gus gave away 7 objects. Today, he has given away 6. How many objects has he given away in the past 2 days?		7 + 6 = ❐
Combination of transformations (negative) Determine the change involved	Yesterday, Gus gave away 7 objects. Today, he has given away more but we don't know how many. If we know that he has given away 13 objects in the past 2 days, how many objects has he given away today?		7 + ❐ = 13
Combination of mixed transformations [2] Determine the gain or the loss	Yesterday, Gus received 7 objects. Today, he has given away 6. How many more or fewer objects does he have after 2 days?		7 – 6 = ❐
Combination of mixed transformations Determine the change involved	Yesterday, Gus received 13 objects. If we know that he has received 7 objects in the past 2 days, how many objects has he received or given away today?		13 – ❐ = 7
Combination of mixed transformations Determine the change involved	Yesterday, Gus gave away 13 objects. If we know that he has received 7 objects in the past 2 days, how many objects has he received or given away today?		–13 + ❐ = 7

[2] Problems involving a mixed combination of transformations require the use of integers. In Elementary Cycle Three, these problems are solved using a diagram or a number line.

Combination of mixed transformations Determine the change involved	Yesterday, Gus received 13 objects. If we know that he has given away 7 objects in the past 2 days, how many objects has he received or given away today?		13 – ❒ = –7

❖ Sense: changes involving addition and subtraction in Cycle One

When working with changes, should we ask students as early as Cycle One to determine the initial state, the changes involved as well as the final state?

Yes, using the appropriate numbers for their cycle. In addition, statement 5 on page 12 of the Progression of Learning indicates that Cycle One students must determine the missing term in an equation (relationships between operations: QEP, p. 151).

$a + b = \square,\ a + \square = c,\ \square + b = c,\ a - b = \square,\ a - \square = c,\ \square - b = c$

Here we see equations in which students must determine the initial state, the change involved and the final state.

References: QEP, p. 151
Progression of Learning, p. 9, no. A-2 a
Progression of Learning, p. 12, no. A-5

❖ Relationships between operations and properties of operations

What contexts can help students develop the concepts of relationships between operations and properties of operations and what is their purpose?

Students are introduced to the concept of relationships between operations as early as Cycle One when they are asked to determine the missing term in equations such as the following:

$$a + b = \blacksquare\ , \quad a + \blacksquare = c,\ \ \blacksquare + b = c,\ \ a - b = \blacksquare,\ \ a - \blacksquare = c,\ \ \blacksquare - b = c$$

In the same way, Cycle Two and Three students develop their understanding of the concept of relationships between operations when they are asked to determine the missing term in equations such as the following:

$$a \times b = \blacksquare, \quad a \times \blacksquare = c, \quad \blacksquare \times b = c, \quad a \div b = \blacksquare, \quad a \div \blacksquare = c, \quad \blacksquare \div b = c$$

Determining numerical equivalencies using relationships between the operation of addition and the *commutative* property of addition in Cycle One ($a + b = b + a$) is a strategy that promotes the mastery of number facts related to addition. Determining numerical equivalencies between the operation of multiplication and the *commutative* property of addition ($a \times b = b \times a$) is also a strategy that promotes the mastery of number facts related to multiplication.

Mental computation also draws on students' operation sense when they effectively use the relationships between operations and their properties. These computation processes are often acquired by transferring their knowledge of the models used when learning operation sense.

The *associative* property of operations is useful when students use a mental computation strategy such as adding *by completing the first term to the next tens place* as in the following example:

$$\begin{aligned} 47 + 14 &= 47 + (3 + 11) \\ &= (47 + 3) + 11 \\ &= 50 + 11 \\ &= 61 \end{aligned}$$

In the same way, the *distributive* property of operations is useful when students use a mental computation strategy such as *multiplying by decomposing the multiplier* (2nd factor):

$$\begin{aligned} 23 \times 12 &= 23 \times (10 + 2) \\ &= (23 \times 10) + (23 \times 2) \\ &= 230 + 46 \\ &= 276 \end{aligned}$$

References: QEP, p. 151
Progression of Learning, p. 9, nos. A-5 a, b, c; p. 10, nos. B-3 a and b; p. 11, no. A-2 b; p. 12 nos. A-6 b, A-5 and A-8

Meaning of operations involving numbers

Decimals

- **Concept of repeated addition**

Why is the concept of repeated addition not included with respect to the multiplication of decimals in the Progression of Learning?

When multiplying a decimal by a natural number, students could choose to perform repeated addition
3.25 X 3 = ?
3.25 + 3.25 + 3.25 = ? "3.25" added 3 times
9.75

Yet, when a decimal is multiplied by another decimal, the concept of repeated addition no longer applies.
3.2 X 3.5 = ?
We could add 3.5 three times, but what should be done with the remaining 0.2?

Reference: Progression of Learning, p. 10, no. B-2

Operations involving numbers

Natural numbers

- **Repertoire of number facts**

1. *Why are there three statements mentioning the repertoire of number facts in the Progression of Learning, while there is only one in the QEP?*

The Progression of Learning is intended to complement the QEP. It provides more specific information about the essential knowledge in the QEP, going into as much detail as possible with a view to outlining the progress that students are supposed to make in this regard.

Students build a repertoire of memorized multiplication and division facts in Cycle Two by **first building a memory of the number facts required in the QEP**, using materials, drawings, charts or tables, and **then** developing strategies that promote mastery of these number facts and relating them to the properties of multiplication (the commutative property of multiplication, identity property and zero product property). After building their memory and developing these strategies, students will **finally** be able to master all the number facts. They need an additional year to consolidate their mastery of this repertoire of memorized facts over the long term.

The same approach applies to the repertoire of memorized addition and subtraction facts

References: QEP, p. 151
Progression of Learning, p. 12, no. A-6 a-b-c, and p. 11, no. A-2 a-b-c

2. *What are students expected to master with respect to the repertoire of memorized addition facts and the corresponding subtraction facts as well as the repertoire of memorized multiplication facts and the corresponding division facts?*

For addition, students memorize number facts from $0 + 0 = 0$ to $10 + 10 = 20$. The corresponding subtractions are $20 - 20 = 0$ to $0 - 0 = 0$. They may do this as a game: for example using the number 8:

$0 + 8 = 8$	$8 + 0 = 8$	$8 - 0 = 8$	$8 - 8 = 0$	
$1 + 8 = 9$	$8 + 1 = 9$	$9 - 1 = 8$	$9 - 8 = 1$	
$2 + 8 = 10$	$8 + 2 = 10$	$10 - 2 = 8$	$10 - 8 = 2$	
$3 + 8 = 11$	$8 + 3 = 11$	$11 - 3 = 8$	$11 - 8 = 3$	
$4 + 8 = 12$	$8 + 4 = 12$	$12 - 4 = 8$	$12 - 8 = 4$	etc.

For multiplication, students memorize number facts from $0 \text{ X } 0 = 0$ to$10 \text{ X } 10 = 100$. The corresponding divisions are $100 \div 10 = 10$ to … (e.g. the 8 times table)

$0 \times 8 = 0$	$8 \times 0 = 0$	$0 \div 8 = 0$	$8 \div 0 =$ Error	
$1 \times 8 = 8$	$8 \times 1 = 8$	$8 \div 8 = 1$	$8 \div 1 = 8$	
$2 \times 8 =16$	$8 \times 2 = 16$	$16 \div 8 = 2$	$16 \div 2 = 8$	
$3 \times 8 = 24$	$8 \times 3 = 24$	$24 \div 8 = 3$	$24 \div 3 = 8$	
$4 \times 8 = 32$	$8 \times 4 = 32$	$32 \div 8 = 4$	$32 \div 4 = 8$	etc.)

References: QEP, p. 151
Progression of Learning, p. 11, no. A-2 and p. 12, no. A-6

- **Own processes and conventional processes**

1*. What are Grade 1 students expected to learn regarding written computation?*

Cycle One students develop processes for written computation using their **own processes** as well as **objects** or **drawings**. They determine the **sum** or the **difference** of two natural numbers **less that 1000**.

References: QEP, p. 151
Progression of Learning, p. 11, no. A-4 a

It is important to remember that students' **own processes are not taught**: they are developed by the students themselves. The teacher's role in helping students develop their own processes is to assign mathematical problems involving different concepts of addition and subtraction (adding, taking away, uniting, comparing and complement), to help students organize their work and to get them to describe what they are doing, which will help them make connections between their own process and the underlying mathematical concept. The teacher should also have students discuss the processes they are using.

2. *Why is it important for students to develop their own processes in elementary school?*

When elementary school students learn to perform operations, they **develop** their own processes to make the operations meaningful. This is a very important step in helping them develop their operation sense. For example, when dividing, students may start by performing repeated subtraction, sharing or using another process to determine a quotient. Then, in Cycle Three, they will be using a conventional process to perform the operation. (By this, we mean a "traditional" long division algorithm). In elementary school, concrete objects or diagrams are used to help students make the transition from using their own processes to implementing conventional processes. Then, in secondary school, the students **carry out** what they learned in elementary school.

References: QEP, p. 157
Progression of Learning, p. 9, no. A-3 b and p. 12, no. A-7c

3. *What is a conventional process?*

A conventional computation process is a **recognized** operational technique that involves applying a series of rules to a set of numbers in a predetermined order to produce a definite result, regardless the numbers used.

- **Using technology (calculator)**

When are calculators used in elementary school?

Both the QEP and the Progression of Learning indicate that "in all cycles, calculators may be used to good advantage as a calculation, verification and learning tool."

They are used:

- to explore natural numbers, decimals, fractions and integers
- to explore operations involving natural numbers, decimals and fractions
- to perform operations involving numbers that go beyond the requirements of the program
- to check calculations
- to solve situational problems consisting of several steps and emphasizing reasoning, research and the implementation of strategies that draw on students' knowledge rather the computation process

Using a calculator is not a conventional computation process.
It can also be helpful to use technological tools other than calculators.

References: QEP, p. 157
Progression of Learning, p. 11, Introduction to the table and p. 12, no. A-15 a-b-c

- **Patterns, series of numbers and families of operations**

What do "families of operations" mean with regard to patterns and series of numbers?

A pattern is a phenomenon that occurs according to a law or a formation rule that applies to a set. A pattern exists when a statement can be made about the properties of a set or when a mathematical formula can be established.
An example of a pattern is a series of numbers established according to a formation rule that makes it possible to determine each term.
Examples:

2, 5, 8, 11, 14, …	the rule is $n + 3$ where n represents the preceding term
1, 2, 4, 8, 16, …	each term in the series is twice the value of the preceding term
3, 2, 5, 4, 7, 6, 9, …	the pattern here is "-1 + 3"
1, 2, 5, 10, 17, … + 1 + 3 + 5 + 7	the pattern here is "+ 1 + 3 + 5 + 7 …"

The following sets are other examples of patterns.

0 00 000 0000	
0 00 000 0000	It is composed successively of 2, 4, 6 and 8 circles.
▲ ■ ● ▲ ■ ●	It is composed of a triangle, a rectangle and an ellipse in succession.

Frieze patterns and tessellations are also examples of patterns.

To develop number and operation sense in this context, students observe different patterns. They recognize and construct patterns, find missing terms in different series, extend them and describe them using words they know or appropriate mathematical language for their cycle. To do so, they determine the operations involved.

The term "family" has several meanings and, among other things, corresponds to a property shared by a given set. It can be associated with several elements. For example, certain reference works express this idea as follows: "family of fractions equivalent to $\frac{2}{3}$: $\frac{4}{6}$, $\frac{6}{9}$, $\frac{8}{12}$."
With regard to patterns, a "family of operations" can be viewed as **the operation or operations common to a series of numbers**. For example, addition and multiplication tables contain many patterns in which we can identify several families of operations because they have common properties linked to one or more operations.
The following are other examples where patterns can be recognized and a common property can be identified:

Multiples of 5:	5, 10, 15, 20, . . .
"one more" starting with a given number	4, 5, 6, 7, . . .
"-3" or take away 3:	14, 11, 8, 5, . . .

The important thing here is not the term "families," but rather that students recognize the pattern.

References: QEP, p. 152
Progression of Learning, p. 12, no. 13

- **Table and table of values**

Could we say that a table of values and a table are the same thing?

No, because a table of values is a tool that helps you visualize the dependency relationship between two elements, whereas a table is used to organize data that do not necessarily have a dependency relationship among them.

References: QEP, p. 154
Progression of Learning, p. 20, nos. 2, 3 a, 3 b, 3 c, 4 a, 4 b and corresponding vocabulary

- **Exponentiation in the order of operations**

Does the statement "Performs a series of operations in accordance with the order of operations" include exponentiation?
Can the exponent be applied to the parentheses as in the example $3 + (5 - 2)^2$?

Since Cycle Three students can calculate the power of a number, exponents may also be included in the series of operations to be performed.
However, applying an exponent to parentheses is asking them to do more than simply calculate the power of a number, but you could include a number with an exponent inside the parentheses, for example $(2 + 3^2)$.

References: QEP, p. 150-151
Progression of Learning, p. 12, nos. A-10 and A-12

Operations involving numbers

Fractions

- **Adding mixed numbers**

Can we ask students to add $2\frac{2}{6}$ *and* $\frac{1}{3}$?

Mixed numbers are a type of fraction and should be covered in elementary school.
In this case, the denominator in the mixed number $2\frac{2}{6}$ is a multiple of the denominator of the fraction $\frac{1}{3}$. Cycle Three students are capable of performing this addition.

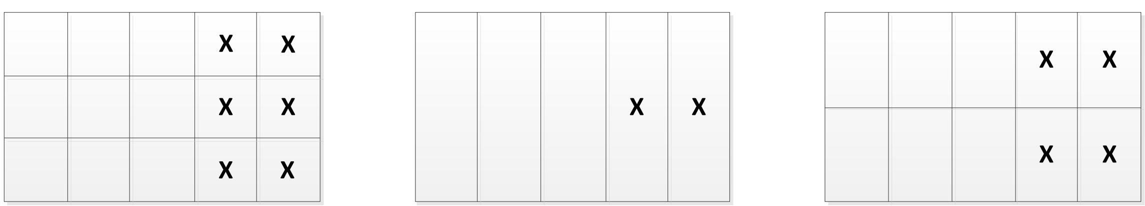
References: QEP, p. 136
Progression of Learning, p. 12, nos. B-1, B-2 and B-3

❖ GCD, LCM and reducing fractions

Are finding the lowest common multiple to perform operations involving fractions and finding the greatest common divisor for reducing fractions still relevant?

Finding the lowest common multiple is useful when adding and subtracting fractions whose denominators are not multiples of each other. In elementary school, operations involving the addition and subtraction of fractions are carried out using concrete objects or diagrams. Cycle Three students only add and subtract fractions where the denominator of one of the fractions is a multiple of the denominator of the other fraction. By finding equivalent fractions using concrete objects or diagrams, students are able to acquire an understanding of the procedure involved in adding and subtracting fractions.

Although the quickest way to determine the irreducible fraction is to divide the numerator and the denominator by their greatest common divisor, Cycle Three students may use concrete material or diagrams to reduce a fraction to its simplest form using the **equivalent fraction** whose numerator and denominator have only 1 as a common divisor, as in the following example.



6/15 2/5 4/10

For each family of equivalent fractions, there is a fraction whose terms no longer have a common divisor other than 1 ($\frac{2}{5}$ in the example above). These are called irreducible fractions.[3]

The terms “lowest common multiple” and “greatest common divisor” are not in the elementary-level QEP or Progression of Learning. In secondary school, these **properties** are used in different contexts to find or produce equivalent expressions and to perform operations involving numbers.

References: QEP, p. 152
Progression of Learning, p. 12, nos. B-2, B-3 and corresponding vocabulary

[1] Roegiers, Xavier, *Les mathématiques à l’école primaire,* Tome 1, p. 163.

Operations involving numbers

Decimals

- **Quotients**

1. *When a natural number is divided by a natural number, isn't the remainder expressed as a fraction?*

Yes, when **Cycle Two students** use their **own processes (with objects or drawings)** to determine the quotient of a three-digit natural number and a one-digit natural number, they express the remainder as a **fraction**.
However, when **Cycle Three students** use **conventional processes** to determine the quotient of a four-digit natural number and a two-digit natural number, they express the answer as a **decimal that does not go beyond the second decimal place**.

Reference: QEP, p. 151
Progression of Learning, p. 12, no. A-7 a and c

2. *What do you do when the quotient resulting from the division of a decimal goes beyond the third decimal place?*

When dividing a decimal by a natural number less than 11, **Cycle Three students** express the quotient **as a decimal without going beyond the second decimal place** **as they do when dividing natural numbers**. When performing the division, they stop when the remainder gets to the second decimal place.

References: QEP, p. 151 and 152
Progression of Learning, p. 13 no. C-3 c and p. 12, no. A-7 c

3. For example: *72.5 divided by 9 = 8.05555555… Do students have to round off to* 8.056 *or* $8.0\overline{5}$ *or do we ensure that they do not encounter this type of situation?*

Cycle Three students can continue their division to the third decimal place and then round their answer off to the second decimal place.

The term "recurring" is not part of the elementary-level vocabulary. The students simply observe that the digit repeats itself.

References: QEP, p.151 and 152
Progression of Learning, p. 13, no. C-3 c, p. 12, no. A-7 c and p. 7, no. C-9

Operations involving numbers

Using numbers

❖ Calculating percentage

1. *How should the concept of percentage be introduced in elementary school?*

In elementary school, developing an understanding of the concept of percentage is the priority. **Determining percentage involves determining equivalent fractions**.

In secondary school, students can use proportional reasoning to calculate "a certain percentage of a number" and to determine "the value corresponding to 100 percent."

References: QEP, p. 150
Progression of Learning:

- p. 7, no. B-9: Matches a decimal or percentage to a fraction
- p. 12, no. B-1: Generates a set of equivalent fractions
- p. 13, no. D-3: Expresses a fraction as a percentage, and vice versa

2. *Do elementary students learn to calculate 10% de 40?*

Elementary students learn to calculate a percentage such as 10% of 40 by using fractional or decimal notation and making connections between what they have learned about fractions and decimals.

$10\% = \frac{10}{100} = \frac{1}{10}$ therefore 10% of 40 is $\frac{1}{10}$ of 40

Geometry

Space

- **Cartesian plane and writing ordered pairs**

How is the Cartesian plane introduced in Elementary Cycle One?

Often, Cycle One students locate objects on an axis or in a plane before being introduced to the Cartesian plane. Students are introduced to the Cartesian plane informally and often in a fun way (e.g. battleship games and chess). These activities are carried out in the **first quadrant** of the Cartesian plane using natural numbers covered in the program. Describing movement on game boards or on chess boards helps familiarize students with the Cartesian reference system and writing coordinates.

References: QEP, p. 152
Progression of Learning, p. 14, nos. A-4a and A-3, p. 6, no. A-10
Progression of Learning, p. 14, A, Symbols

Solids

- **Describing solids**

Which solids are described in terms of faces, vertices and edges?

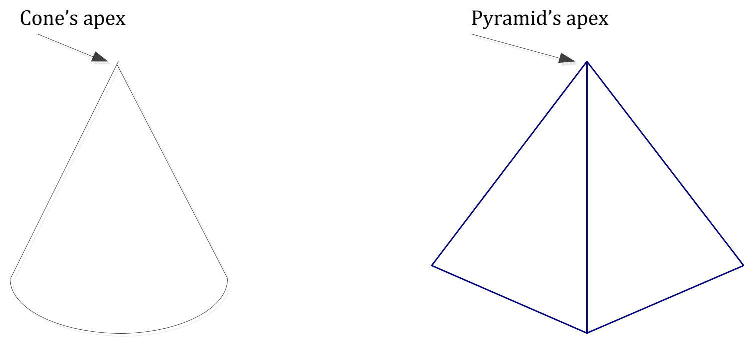
Elementary Cycle One students **compare**, **construct** and **identify** spheres, cones, cubes, cylinders, prisms and pyramids. They **identify** and **represent the faces** of prisms and pyramids. Cycle Two students **describe** prisms and pyramids in terms of **faces**, **vertices** and **edges**. Elementary students are not expected to describe curved body in terms of faces, vertices and edges: they do this only with prisms and pyramids.

References: QEP, p. 152
Progression of Learning, p. 14, nos. B-2 and B-3, p. 15, nos. B-4 and B-5

- ❖ **Cone: apex or vertex?**

Since the term apex does not appear in the QEP or in the Progression of Learning for either elementary or secondary school, do cones (or pyramids) have vertices?

Apex is the term used for certain notable vertices (vertex at the tip of a cone or pyramid, vertex opposite the base in a triangle).



However, in elementary Cycle One, the Progression of Learning states that students compare, construct and identify solids. They observe the base, faces and flat or curved surfaces of solids. **Students are not expected to describe the solids in terms of their apex, directrix and generatrix**.

When students reach Cycle Two, they begin to describe prisms and pyramids in terms of faces, vertices and edges.

References: QEP, p. 136
Progression of Learning, p. 14, nos. B-2, B-3 and corresponding vocabulary, p. 15, no. B-5

Plane figures

- ❖ **Constructing plane figures**

If students cannot use their protractors to draw angles, how can they construct perpendicular lines and consequently geometric figures?

Could we ask students to construct a square to evaluate the construction of parallel and perpendicular lines?

Students start to identify and **construct** parallel and perpendicular lines in Cycle Two. To do so, they use various construction strategies involving, for example, rulers, set squares, graph paper or dotted paper. In this way, they are able to draw right angles (perpendicular lines) and parallel lines from which they can construct squares and rectangles. In this context, using a protractor to draw a right angle is not necessary. Once they have developed these construction strategies in class, it is possible to evaluate their ability to draw parallel and perpendicular lines by asking them to construct a figure such as a square, but you must take into consideration the fact that this task is more complex. Other contexts can be used to determine whether students have grasped these concepts.

Students learn to *Compare and construct figures made with closed curved lines or closed straight lines* in Cycle One. This statement refers to **developing a representation of figures** where the figures are constructed imprecisely, by hand or using graph paper.

References: QEP, p. 152-153
Progression of Learning, p. 15, nos. C-1 and C-5

Measurement

Lengths

- **Constructing rulers**

What does the expression "constructs rulers" in the Progression of Learning mean? Are students expected to construct real rulers or rulers using unconventional units?

The Essential Knowledges section in the QEP states the following with regard to Cycle One:

Unconventional units: comparison, **construction of rulers**

Constructing rulers helps students develop measurement sense. Students start by designing, constructing and using measuring instruments with unconventional units. They then construct and use conventional measuring instruments and work with units of measure that are appropriate for their cycle.

References: QEP, p. 137
Progression of Learning, p. 17, nos. A-2 and A-4 a

Surface areas and Volumes

- **Surface area and volume**

In elementary school, do students use their own methods to calculate area and volume?

Elementary school teachers do not teach the relations (formulas) for calculating area or volume. Consequently, students use other processes to **measure** area or volume. Some students discover **on their own** that measuring the area of a rectangle involves the same process as multiplication and thus use their understanding of *rectangular arrays*. Others use their understanding of *repeated addition*, while some count the square units on the surface one by one.

In Secondary Cycle One, students **construct relations** (formulas) to **calculate** the area of plane figures.

References (elementary): QEP, p. 153
Progression of Learning, p. 18, nos. B-1 b and C-1 b

References (secondary): QEP, Mathematics, Secondary Cycle One, p. 216
Progression of Learning in Secondary School, p. 31, no. E-4

Angles

❖ Constructing angles

When do students learn to construct angles?

Elementary Cycle Two students **compare angles**: right angles, acute angles and obtuse angles. Cycle Three students **estimate and determine the degree measurement of angles** using their protractors. With regard to plane figure geometry, Cycle Two students identify and **construct** parallel lines and **perpendicular lines** (right angles).

Using a protractor to construct acute and obtuse angles is not covered in elementary school.

References: QEP, p. 153
Progression of Learning, p. 18, nos. D-1, D-2 and corresponding vocabulary, p.15, no. C-5

Time

❖ Measuring time

How do you teach students to measure time and, more particularly, to calculate durations?

In Cycle One, students estimate and measure time using the following units of measure: *day*, *hour*, *minute* and *second*. They learn to read time and its representations (e.g. 03:25, 3:25 a.m.) and they calculate **simple** durations and time intervals (e.g. the duration of recess from 10:15 to 10:30 or the duration of music class from 10:30 to 11:30).

In Cycle Two, students also learn the following units of measure: *daily cycle*, *weekly cycle*, and *yearly cycle*. Students estimate and measure **more complex** time intervals (e.g. the duration of a bus trip from 3:50 p.m. to 5:07 p.m., the duration of an appointment from 3:20 p.m. to 5:45 p.m.). It is only at the end of Cycle Two that student estimate and measure time on their own using conventional units of measure.

Throughout the three cycles of elementary school, students establish relationships among the units of measure that are appropriate for their cycle. Students only begin to establish relationships among the different units of measure on their own at the end of Cycle Three.

References: QEP, p. 153
Progression of Learning, p. 19, nos. G-1, G-2 and corresponding vocabulary

Statistics

- **Concept of mean in elementary and secondary school**

What is the difference between the mean covered in elementary school and the mean covered in Secondary Cycle One?

In elementary school, the numbers used to calculate the arithmetic mean are natural numbers and positive decimals. In addition, when using **written computation** to determine the mean of decimals, the problem cannot contain more than 10 data values; if the problem involves more than 10 values, then a calculator or another form of technology should be used. Students understand the concept (represent the concept using materials to demonstrate levelling) and apply it to perform their calculations.

The operations studied in Secondary Cycle One involve positive fractions and positive and negative decimals. There is no restriction regarding the number of data values. Students must understand and calculate mean and be able to describe the concept. They are also asked to **interpret**. For example, students may choose to use the mean to interpret and analyze the results of a distribution.

References (elementary):	QEP, p. 154 Progression of Learning, p. 20, no. 5 and p. 13, no. C-3 c
References (secondary):	QEP, Mathematics, Secondary Cycle One, p. 215 Progression of Learning in Secondary School, p. 25, nos. 9 and 10