Learning Material Covered in Each Elementary School Cycle

Arithmetic

Understanding and writing numbers, pp. 5, 6, 7 and 8

A. Natural numbers, pp. 5-6

Cycle One	Cycle Two	Cycle Three	Reminder
Numbers covered: less than 1000	Numbers covered: less than 100 000	Numbers covered: less than 1 000 000	As they move from one cycle to the next, students add to their repertoire of natural numbers studied, which they will use to perform the mathematical actions indicated in nos. A-1 to A-13, pp. 5-6.
p. 5, no. A-2 Counts collections (using objects or drawings) c. counts a collection by grouping or regrouping	p. 5, no. A-2 Counts collections (using objects or drawings) c. counts a collection by grouping or regrouping d. counts a pre-grouped collection	p. 5, no. A-2 Counts collections (using objects or drawings) c. counts a collection by grouping or regrouping d. counts a pre-grouped collection	Cycle One students count collections by forming groups that have meaning for them (e.g. groups of 3, 5 or 2, not necessarily groups of 10). Only in Cycle Two do students start counting collections that are already divided into groups that may be less meaningful to them (e.g. models based on equivalency).
p. 5, no. A-4 Represents natural numbers in different ways or associates a number with a set of objects or drawings	p. 5, no. A-4 Represents natural numbers in different ways or associates a number with a set of objects or drawings	p. 5, no. A-4 Represents natural numbers in different ways or associates a number with a set of objects or drawings	Place value tables are used to understand number systems where emphasis is placed as much on exchange as on place value.
p. 6, no. A-5 Composes and decomposes a natural number in a variety of ways	p. 6, no. A-5 Composes and decomposes a natural number in a variety of ways	p. 6, no. A-5 Composes and decomposes a natural number in a variety of ways	Elementary school students do not decompose natural numbers using base 10 powers. e.g.: $4\,123 = 4 \times 10^3 + 1 \times 10^2 + 2 \times 10 + 3 \times 1$

Learning Material Covered in Each Elementary School Cycle

B. Fractions (using objects or drawings), pp. 6-7			
Cycle One	**Cycle Two**	**Cycle Three**	**Reminder**
p. 6, no. B-1 Identifies fractions related to everyday items (using objects or drawings) p. 6, no. B-2 Represents a fraction in a variety of ways, based on a whole or a collection of objects	p. 6, no. B-2 Represents a fraction in a variety of ways, based on a whole or a collection of objects p. 7, no. B-3 Matches a fraction to part of a whole (congruent or equivalent parts) or part of a group of objects, and vice versa p. 7, no. B-6 Reads and writes a fraction p. 6, no. B-5 Distinguishes a numerator from a denominator p. 7, no. B-4 Identifies the different meanings of fractions (sharing, division, ratio)	p. 6, no. B-2 Represents a fraction in a variety of ways, based on a whole or a collection of objects p. 7, no. B-4 Identifies the different meanings of fractions (sharing, division, ratio)	Cycle One students recognize the fractions one half, one third and one quarter in their environment. They represent these fractions using concrete objects or diagrams associated with daily life. Starting in Cycle Two, students explore other fractions (e.g. one tenth, one hundredth, one fifth, three tenths, two thirds, three quarters) Only in Cycle Two do students begin matching fractions to part of a whole or to a collection. There is a difference between "representing" a fraction (showing it using concrete objects or drawings) and "matching" a fraction to a part of a whole or to a collection (connecting the representation of the fraction to its corresponding fractional notation $\frac{a}{b}$). In Cycle Two, by reading and writing fractions (fractional notation $\frac{a}{b}$) and learning about the different meanings of fractions, students are able to distinguish between the roles of the numerator and the denominator. **With the help of their teacher**, Cycle Two and Three students learn to recognize the different meanings of fractions. It is important to assign problems that reflect these different meanings.

Learning Material Covered in Each Elementary School Cycle

Cycle One	Cycle Two	Cycle Three	Reminder
	Second year of the cycle (Grade 4) p. 7, no. B-9 Matches a decimal or a percentage to a fraction	p. 7, no. B-9 Matches a decimal or a percentage to a fraction	In grade 4, only everyday objects or situations are used when matching a decimal or a percentage to a fraction. This should be viewed merely as an **initiation**, since the concept of percentage is developed only in Cycle Three.
	p. 7, no. B-7 Compares a fraction to 0, $\frac{1}{2}$ or 1 **Second year of the cycle (Grade 4)** p. 7, no. B-10 Orders fractions with the same denominator	**First year of the cycle (Grade 5)** p. 7, no. B-10 Orders fractions with the same denominator **Cycle Three** p. 7, no. B-11 Orders fractions where one denominator is a multiple of the other(s) p. 7, no. B-12 Orders fractions with the same numerator p. 13, no. D-1 Expresses a decimal as a fraction, and vice versa	Ordering involves comparing numbers. Students must be able to compare two fractions before ordering a set of fractions. In grade 4, students are **introduced** to the concept of ordering fractions using fractions with the same denominator. Cycle Two students who examine the concept of two equivalent fractions by writing each fraction with the same denominator (see p. 7, no. B-8) can also order a set of fractions that have the same denominator. Students compare and order fractions (same notation). However, the ability to express a fraction as decimal and vice versa enables Cycle Three student to include decimals in a set of fractions to be ordered (see p. 13 no. D-1).

Learning Material Covered in Each Elementary School Cycle

C. Decimals up to . . . ,pp. 7-8			
Cycle One	**Cycle Two**	**Cycle Three**	**Reminder**
Decimals are not covered in Cycle One.	Numbers covered: up to the second decimal place	Numbers covered: up to the third decimal place	As they move from one cycle to the next, students add to their repertoire of decimals studied, which they will use to perform the mathematical actions indicated in nos. C-1, C-2, C-3, C-5, C-6, C-7, C-8, C-9, C-10 and C-11 on pp. 7-8.
D. Integers, p. 8			
Cycle One	**Cycle Two**	**Cycle Three**	**Careful**
Integers are not covered in Cycle One.	p. 8, no. D-1 **Second year of the cycle (Grade 4)** Represents integers in a variety of ways (using objects or drawings) (e.g. tokens in two different colours, number line, thermometer, football field, elevator, hot air balloon)	p. 8, no. D-1 Represents integers in a variety of ways (using objects or drawings) (e.g. tokens in two different colours, number line, thermometer, football field, elevator, hot air balloon)	Grade 4 students are **introduced** to the concept of representing an integer using everyday situations that involve objects or drawings. Students are asked to represent, locate on a number line, compare and order positive integers (natural numbers) and their opposites as well as negative integers. They do not work with negative rational numbers (negative fractions and decimals).

Learning Material Covered in Each Elementary School Cycle

Meaning of operations involving numbers, pp. 9-10			
A. Natural numbers, pp. 9-10			
Cycle One	**Cycle Two**	**Cycle Three**	**Careful**
Numbers covered: less than 1000	Numbers covered: less than 100 000	Numbers covered: less than 1 000 000	As they move from one cycle to the next, students add to their repertoire of natural numbers studied, which they will use to perform the mathematical actions indicated in nos. A-1 to A-6, p. 9.
p. 9, no. A-2 Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of addition and subtraction)	p. 9, no. A-2 Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of addition and subtraction)	p. 9, no. A-2 Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of addition and subtraction)	Using all these concepts of addition and subtraction is essential. Students should develop their own representations of these structures, but are not required to know what they are called.
p. 9, no. A-3 Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of multiplication and division) N.B. (using objects and drawings)	p. 9, no. A-3 Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of multiplication and division) N.B. (using concrete objects, diagrams or equations)	p. 9, no. A-3 Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of multiplication and division) N.B. (using concrete objects, diagrams or equations)	In Cycle One, students only use objects or drawings to represent situations (concepts s of multiplication and division). Only in Cycle Two do students begin to use equations to represent situations. Using all these concepts of multiplication and division is essential. Students should develop their own representations of these structures, but are not required to know what they are called.
		p. 9, no. A-6 Translates a situation using a series of operations in accordance with the order of operations	Order of operations is introduced in Cycle Three using simple sequences of operations with only one level of parentheses.

Learning Material Covered in Each Elementary School Cycle

A. Decimals up to . . . ,p. 10			
Cycle One	**Cycle Two**	**Cycle Three**	**Careful**
Since decimals are not covered in Cycle One, they are not used to represent situations.	Numbers covered: up to the second decimal place	Numbers covered: up to the third decimal place	As they move from Cycle Two to Cycle Three, students add to their repertoire of decimals studied, which they will use to perform the mathematical actions indicated in nos.B-1 to B-4, p. 10.
		p. 10, no. B-4 Translates a situation into a series of operations in accordance with the order of operations	Cycle Three students represent situations using simple sequences of operations (only one level of parentheses) containing decimals, but they do not use processes for **written computation**∗ to solve the sequence of operations. ∗*They may use technology (calculator).*
C. Fractions, p. 10			
Cycle One	**Cycle Two**	**Cycle Three**	**Careful**
Fractions are not used to represent situations in Cycles One and Two.		p. 10, no. C-1 Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of addition, subtraction and multiplication by a natural number)	Students do not multiply and divide fractions until secondary school.
Integers			
Integers are not used to represent situations in Cycles One, Two and Three.			In elementary school, students simply develop their understanding of integers and learn to write them.

Learning Material Covered in Each Elementary School Cycle

Operations involving numbers, pp. 11, 12 and 13

A. Natural numbers, pp. 11-12

Cycle One	Cycle Two	Cycle Three	Careful
Numbers covered: less than 1000	Numbers covered: less than 100 000	Numbers covered: less than 1 000 000	As they move from one cycle to the next, students add to their repertoire of natural numbers studied, which they will use to perform the mathematical actions indicated in nos. A-1, A-3, A-9, A-10, A-11 and A-13, pp. 11-12.
p. 11, no. A-2 Builds a repertoire of memorized* addition and subtraction facts a. Builds a memory of addition facts (0 + 0 to 10 + 10) and the corresponding subtraction facts, using objects, drawings, charts or tables *terms less than 11 b. Develops various strategies that promote mastery of number facts and relates them to the properties of addition c. Masters all addition facts (0 + 0 to 10 + 10) and the corresponding subtraction facts	p. 11, no. A-2 **First year of Cycle Two (Grade 3)** Builds a repertoire of memorized addition and subtraction facts b. Develops various strategies that promote mastery of number facts and relates them to the properties of addition c. Masters all addition facts (0 + 0 to 10 + 10) and the corresponding subtraction facts		Strategies that promote the mastery of number facts related to the properties of addition are: • the commutative property of addition 3 + 2 = 2 + 3 • additive identity property 0 + 2 = 2 2 – 0 = 2 Only by the end of grade 3 have students developed strategies to master the number facts related to addition and the corresponding subtractions as well as mastered all addition and subtraction facts.

Learning Material Covered in Each Elementary School Cycle

Cycle One	Cycle Two	Cycle Three	Careful
	p. 12, no. A-6 Builds a repertoire of memorized multiplication and division facts a. Builds a memory of multiplication facts (0 ×0 to 10 × 10) and the corresponding division facts, using objects, drawings, charts or tables b. Develops various strategies that promote mastery of number facts and relates them to the properties of multiplication c. Masters all multiplication facts (0 × 0 to 10 × 10) and the corresponding division facts	p. 12, no. A-6 **First year of the cycle (Grade 5)** Builds a repertoire of memorized multiplication and division facts b. Develops various strategies that promote mastery of number facts and relates them to the properties of multiplication c. Masters all multiplication facts (0 × 0 to 10 × 10) and the corresponding division facts	Strategies that promote the mastery of number facts related to the properties of multiplication are: • the commutative property of multiplication ($3 \times 2 = 2 \times 3$) • multiplicative identity property ($3 \times 1 = 3$) • zero product property ($0 \times 3 = 0$) Only by the end of grade 5 have students developed strategies to master the number facts related to multiplication and the corresponding divisions as well as mastered all multiplication and division facts.
p. 11, no. A-3 Develops processes for mental computation	p. 11, no. A-3 Develops processes for mental computation	p. 11, no. A-3 Develops processes for mental computation	In each cycle, students develop processes for mental computation using the numbers appropriate for their cycle. The computations must be "doable" using the mental computation strategies developed by the students.
	p. 12, no. A-7 Develops processes for written computation (multiplication and division)	p. 12, no. A-7 Develops processes for written computation (multiplication and division)	The concepts of multiplication and division are developed in Cycle One by using objects and diagrams (see p. 9, no. A-3 a) to represent situations, but the students are not required to develop their own written computation processes for these operations.

Learning Material Covered in Each Elementary School Cycle

Cycle One	Cycle Two	Cycle Three	Careful
p. 12, no. A-13 Using his/her own words and mathematical language that is at an appropriate level for the cycle, describes: a. non-numerical patterns (e.g. series of colours, shapes, sounds, gestures) b. numerical patterns (e.g. number rhymes, tables and charts) c. series of numbers and family of operations	p. 12, no. A-13 Using his/her own words and mathematical language that is at an appropriate level for the cycle, describes: c. series of numbers and family of operations	p. 12, no. A-13 Using his/her own words and mathematical language that is at an appropriate level for the cycle, describes: c. series of numbers and family of operations	Cycle One students describe non-numerical and numerical patterns. Non-numerical patterns can be observed in frieze patterns and tessellations (see p. 16, no. D-2). The term "family" corresponds, among other things, to a property shared by a given set. For example, "family of fractions equivalent to $\frac{2}{3} : \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots$" With regard to patterns, a "family of operations" is viewed as **the operation or operations common to a series of numbers**. Examples: • Addition and multiplication facts consist of many patterns. Several families of operations can be identified in these patterns, since they share common properties related to one or more operations. • Multiples of 5. • "One more" starting with a given number • "-3" can be observed in 14, 11, 8, 5. The important thing is not the term "family," but rather that students recognize the pattern.

Learning Material Covered in Each Elementary School Cycle

B. Fractions (using objects or diagrams), p. 12			
Cycle One	**Cycle Two**	**Cycle Three**	**Careful**
Operations involving fractions are not covered in Cycle One.		p. 12, no. B-3 Adds and subtracts fractions when the denominator of one fraction is a multiple of the other fraction(s) p. 12, no. B-4 Multiplies a natural number by a fraction	The multiplication and division of fractions is not covered in elementary school.
C. Decimals, p. 13			
Cycle One	**Cycle Two**	**Cycle Three**	**Careful**
Operations involving decimals are not covered in Cycle One, because decimals are not studied in Cycle One.	Numbers covered: up to the second decimal place	Numbers covered: up to the third decimal place	As they move from Cycle Two to Cycle Three, students add to their repertoire of decimals, which they will use to perform the mathematical actions indicated in nos. C-1 to C-3, p. 13.
	p. 13, no. C-2 Develops processes for mental computation	p. 13, no. C-2 Develops processes for mental computation	In each cycle, students develop processes for mental computation using the numbers appropriate for their cycle. The computations must be “doable” using the mental computation strategies developed by the students.
	p. 13, no. C-3 Develops processes for written computation	p. 13, no. C-3 Develops processes for written computation	Even if students understand decimals up to the third decimal place and are able to write them, their answer must not go beyond the second decimal place when they perform computations that involve the addition, subtraction, multiplication and division of decimals.

Learning Material Covered in Each Elementary School Cycle

Integers			
In Cycles One, Two and Three, integers are not used when performing mathematical operations.			
D. Using numbers, p. 13			
Cycle One	**Cycle Two**	**Cycle Three**	**Careful**
			Matching a fraction to its decimal (see p. 7, no. C-11) is not the same thing as **expressing** a decimal as a fraction: When I **match,** I am presented with numbers that I must *relate to each other*. When I **express,** I *convert* a number from one form to another and *write* it down.